

Characterization of complex and “microstructured” media using elastic guided waves

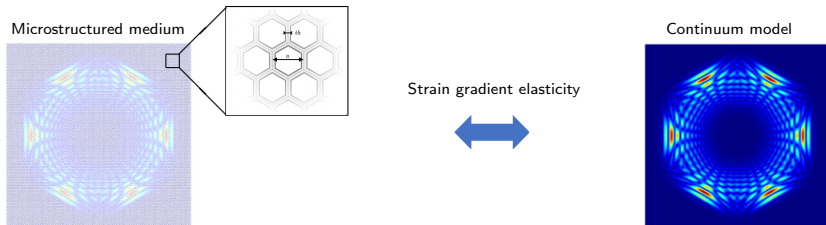
Nicolas Bochud

Université Paris-Est Créteil, CNRS (UMR 8208),
Laboratoire Modélisation et Simulation Multi Echelle,
Biomechanics group

Workshop on elastodynamics of microstructured media,
École des Ponts – ParisTech – Champs sur Marne

Motivation

Continuum modeling of frequency dependent acoustic beam in hexagonal lattices



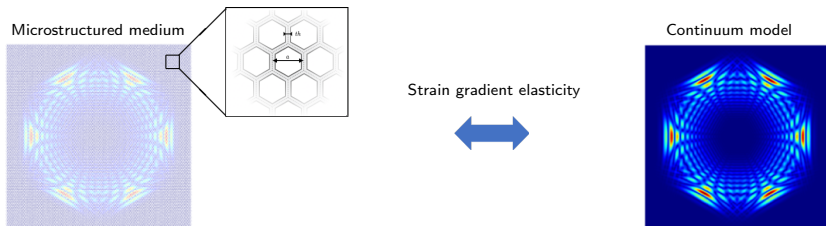
Identification of the constitutive law of strain gradient elasticity models

$$\begin{pmatrix} p \\ q \\ \gamma \\ \gamma \\ \gamma \\ \gamma \end{pmatrix} = \begin{pmatrix} \rho I & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 \\ 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & A \\ \gamma & \gamma & \gamma & \gamma \end{pmatrix} \begin{pmatrix} \nabla \cdot v \\ \nabla \cdot v \\ \gamma \\ \gamma \end{pmatrix}$$

[1] Auffray et al, *Int J Solids Struct*, 2015; [2] Rosi and Auffray, *Eur J Mech A-Solid*, 2019.

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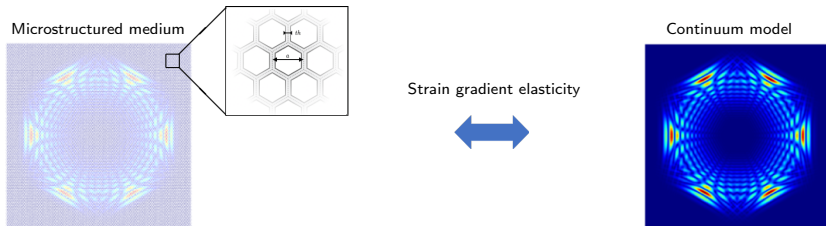
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- Identification of the model parameters (a_S , a_P , a_D , J_P , J_S)?
- Influence of the unit cell characteristics (a , th)?

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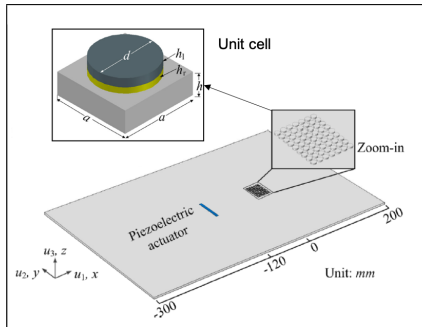
- Identification of the model parameters (a_S, a_P, a_D, J_P, J_S)?
- Influence of the unit cell characteristics (a, th)?

Can we take advantage of guided waves measurements to retrieve such parameters ?

Background and objective

Elastic guided waves in media with surface structures

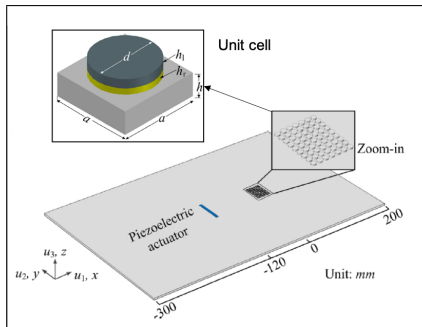
Guided waves focusing using metamaterials



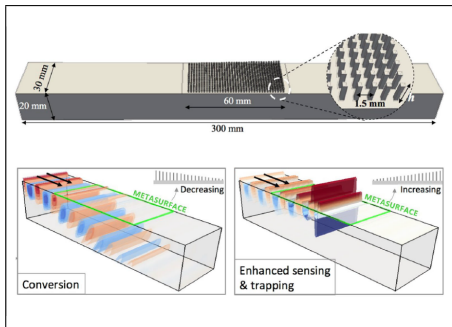
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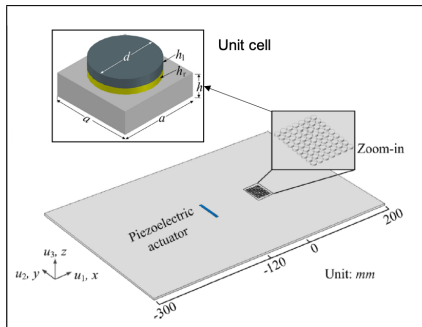
Rayleigh waves conversion using metasurfaces



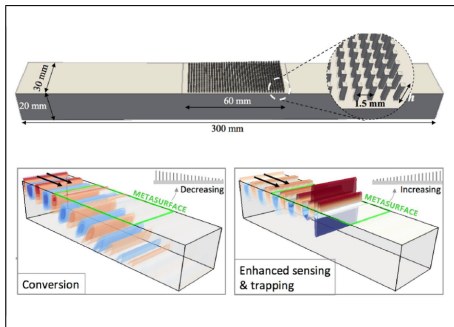
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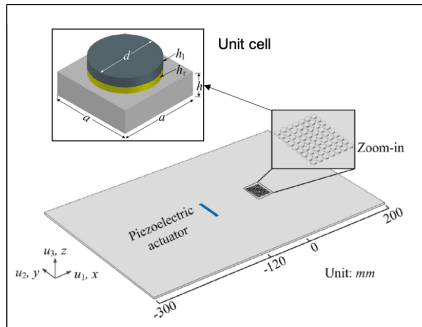


- Little work has been done using micro-architected media **through-the-thickness**
- Can we use **recent experimental advances** as a starting point ?

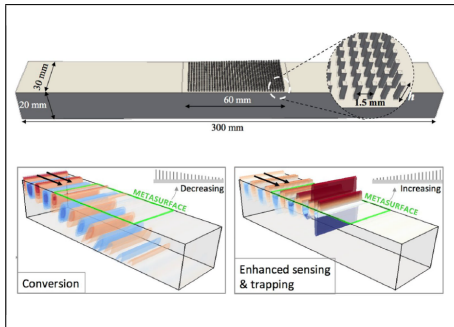
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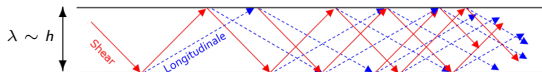
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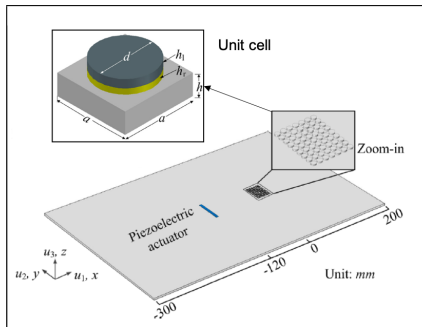
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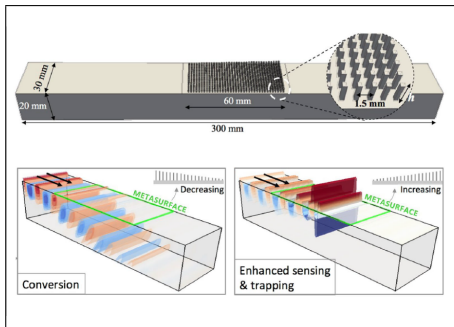
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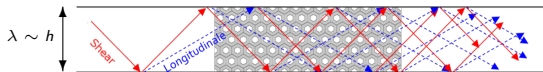
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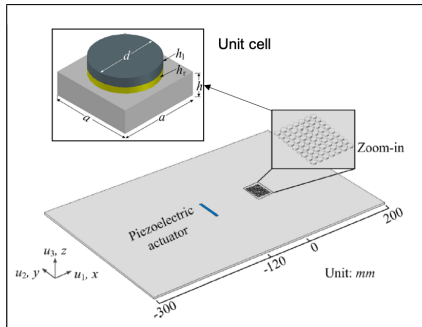
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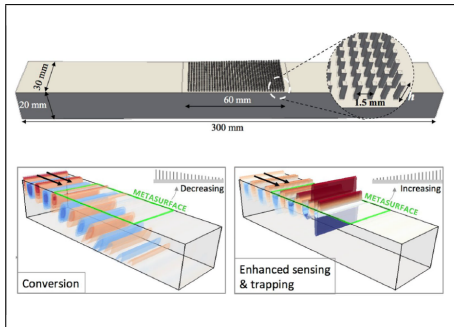
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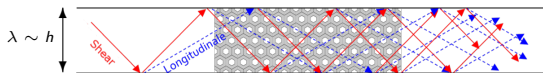
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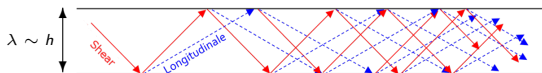


- Guided wave measurements
- Waveguide models
- Inverse problem

Theoretical reminder on guided waves

Elastic guided waves in a homogeneous and isotropic solid

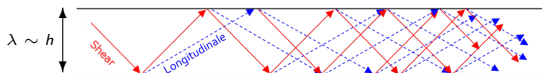
- Modes conversion at the interfaces ($\lambda \sim h$)



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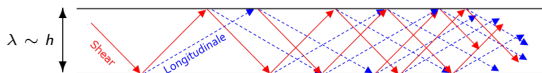
- Rayleigh-Lamb equation

$$\frac{\omega^4}{V_T^4} = 4k^2 q^2 \left[1 - \frac{p \tan(ph + \alpha)}{q \tan(qh + \alpha)} \right], \text{ avec } p^2 = \frac{\omega^2}{V_L^2} - k^2, q^2 = \frac{\omega^2}{V_T^2} - k^2, \alpha = \{0, \pi/2\}$$

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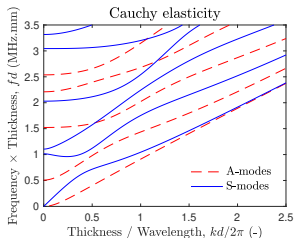
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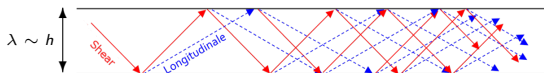
- Solutions: dispersion curves $f(k)$



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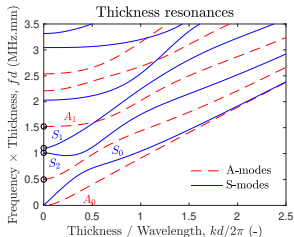
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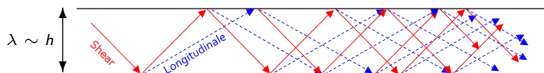
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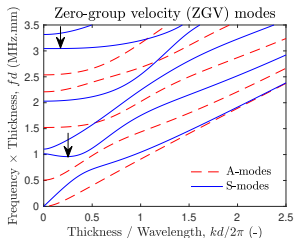
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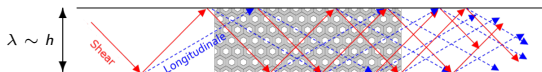
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Theoretical reminder on guided waves

Elastic guided waves in a **microstructured** solid

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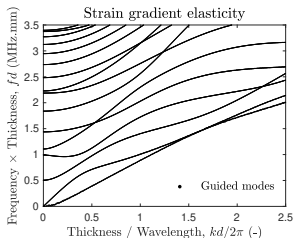
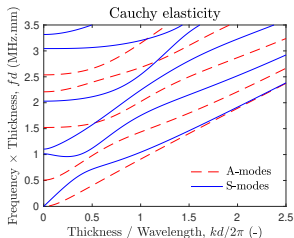


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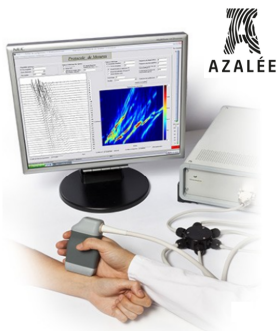
+ $\{a_{11}, a_{12}, a_{22}, a_{23}, a_{44}, J_P, J_S\}$



Overview of recent experimental advances

Guided waves characterization of complex media with “microstructures”

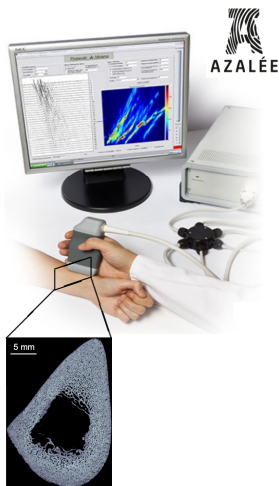
1. Cortical bone assessment



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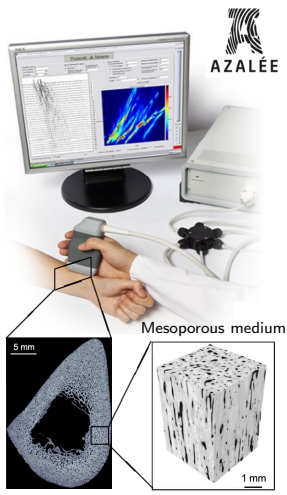
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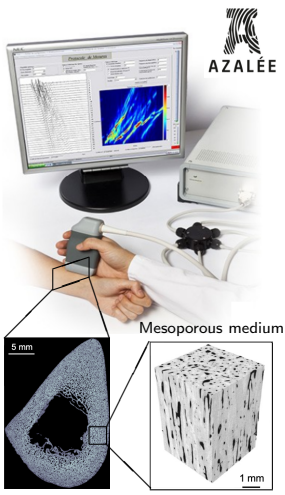
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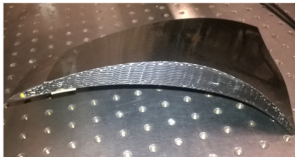
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2. Adhesion of bonded layers

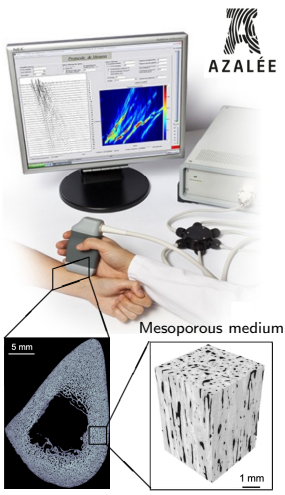


Safran Tech

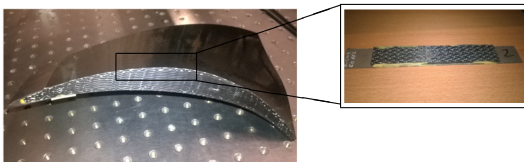
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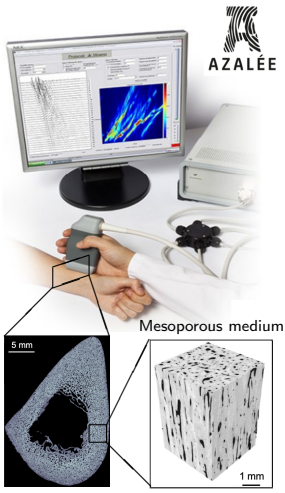


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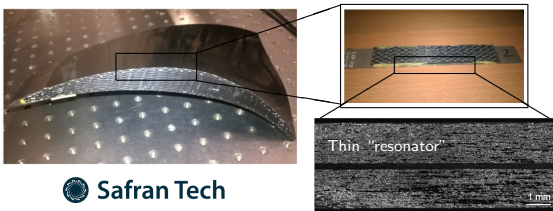
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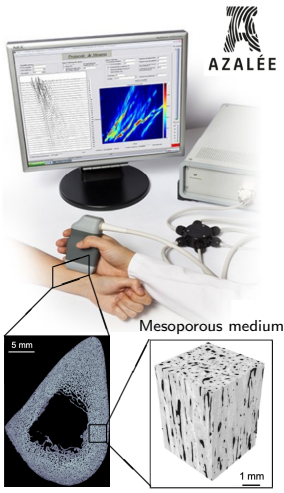
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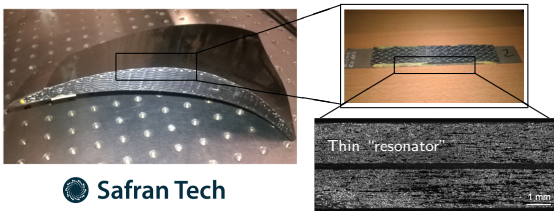
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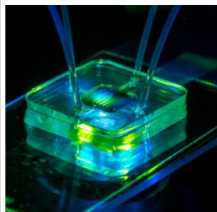
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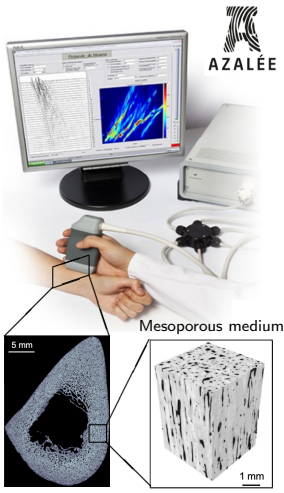
3. Nanoporous silicon membranes



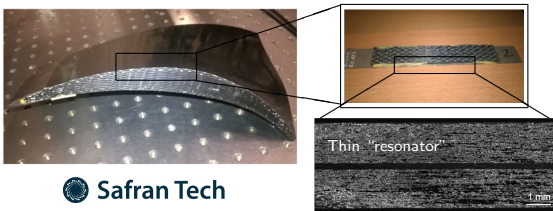
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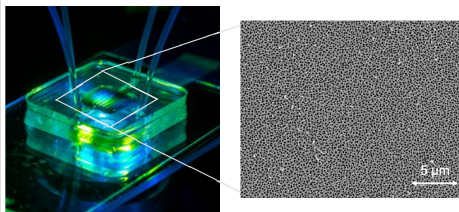
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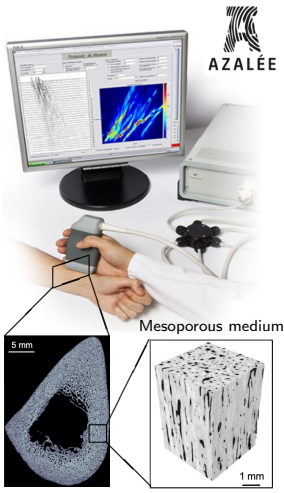
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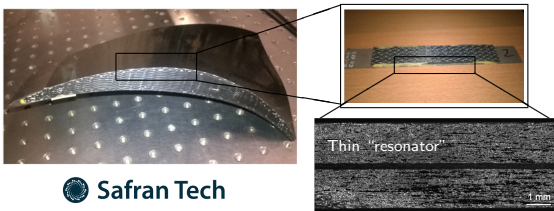
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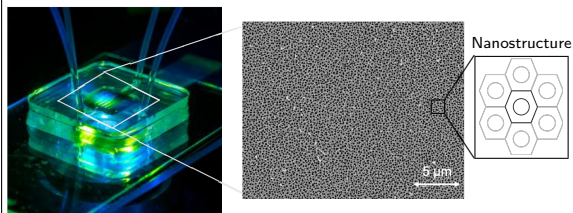
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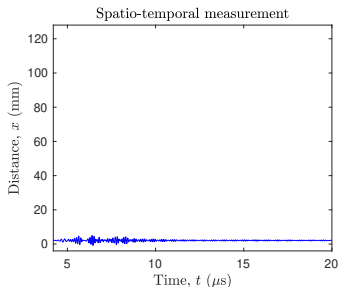
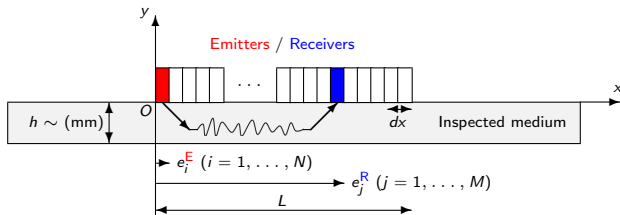


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Measurements of multimode dispersion curves

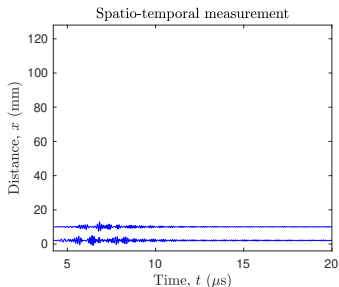
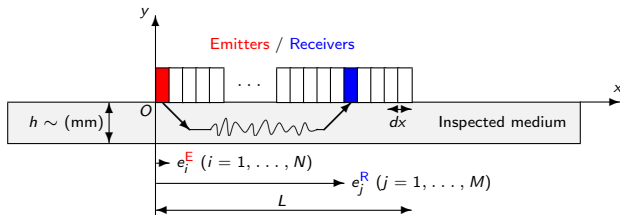
I. Multi-elements probe driven by a programmable multi-channel electronics



[1] Minonzio et al, *J Acoust Soc Am*, 2010; [2] Alleyne and Cawley, *J Acoust Soc Am*, 1991.

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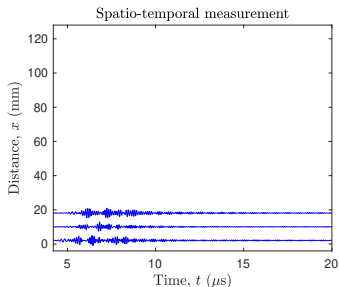
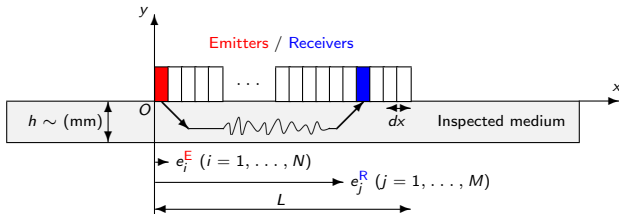
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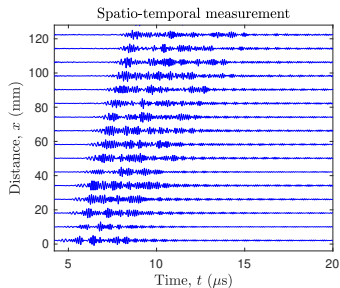
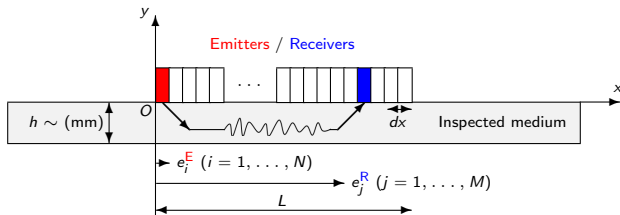
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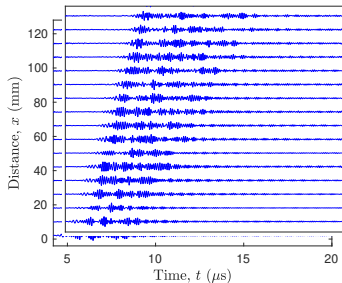
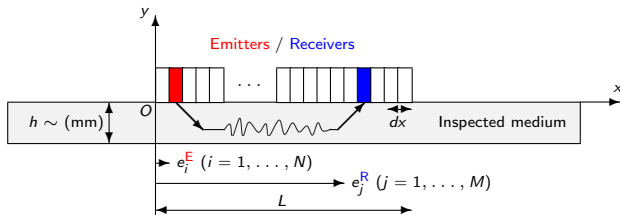
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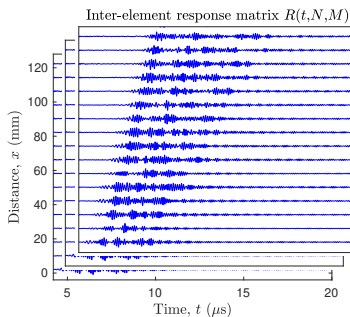
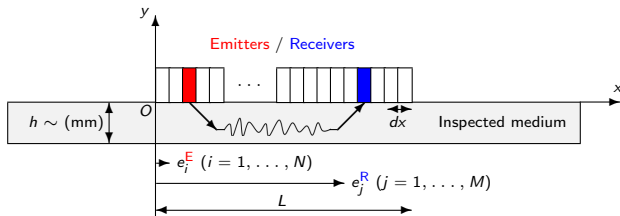
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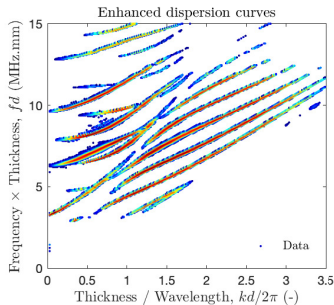
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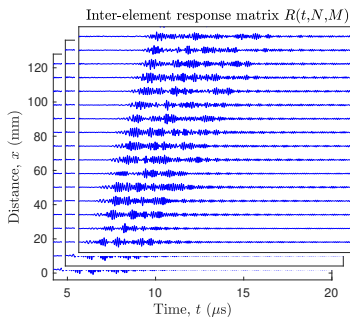
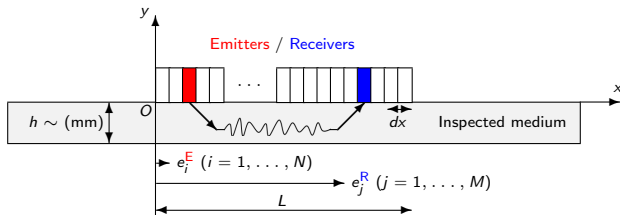


2D-FFT + SVD

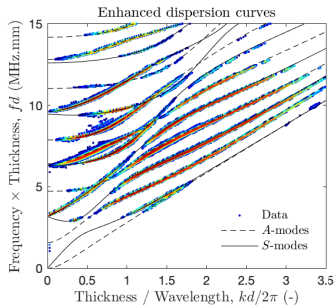


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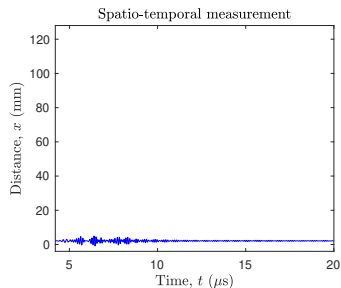
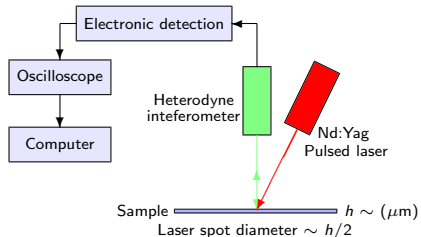


2D-FFT + SVD



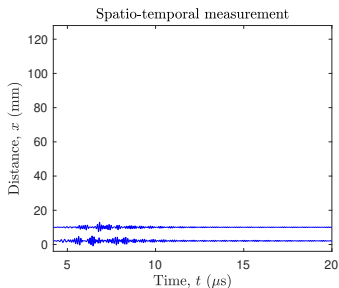
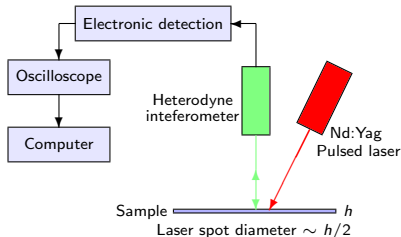
Measurements of multimode dispersion curves

II. Non-contact laser ultrasound measurements



Measurements of multimode dispersion curves

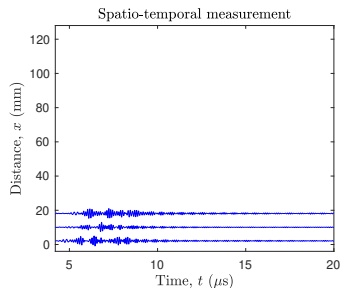
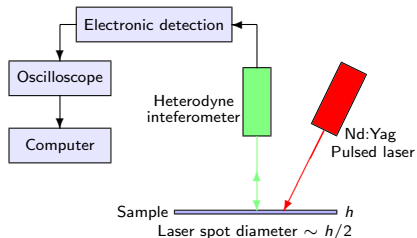
II. Non-contact laser ultrasound measurements



[1] Clorenec et al, *Appl Phys Lett*, 2006; [2] Ces et al., *J Acoust Soc Am*, 2012.

Measurements of multimode dispersion curves

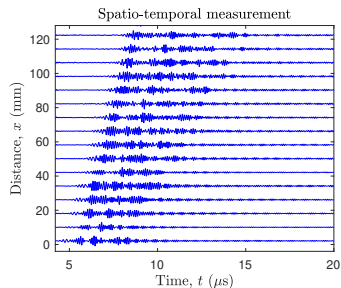
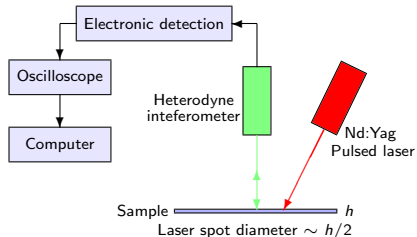
II. Non-contact laser ultrasound measurements



[1] Clorenec et al, *Appl Phys Lett*, 2006; [2] Ces et al., *J Acoust Soc Am*, 2012.

Measurements of multimode dispersion curves

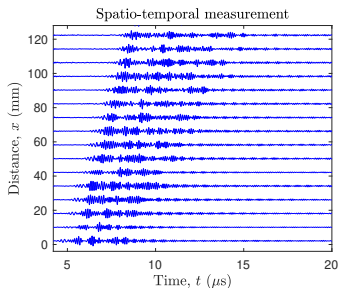
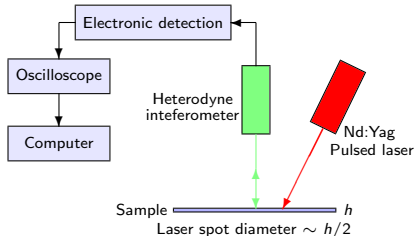
II. Non-contact laser ultrasound measurements



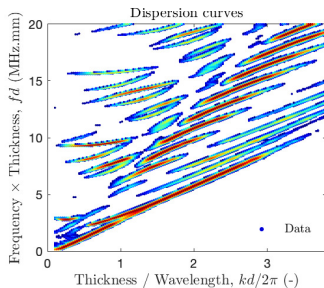
2D-FFT
→

Measurements of multimode dispersion curves

II. Non-contact laser ultrasound measurements

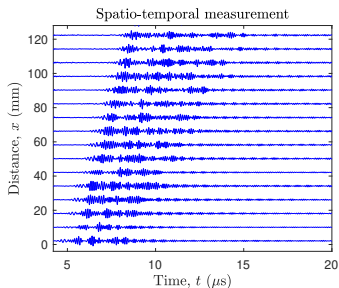
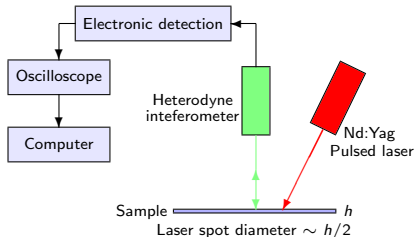


2D-FFT
→

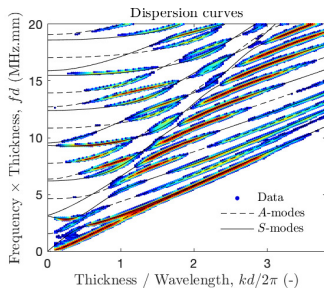


Measurements of multimode dispersion curves

II. Non-contact laser ultrasound measurements



2D-FFT
→

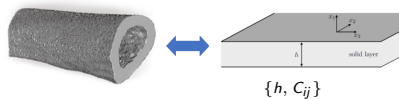


[1] Clorenec et al, *Appl Phys Lett*, 2006; [2] Ces et al., *J Acoust Soc Am*, 2012.

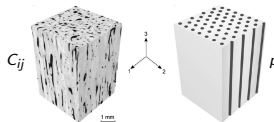
Waveguide models: solving the forward problem

1. Cortical bone assessment

Hypothesis 1: 2-D transverse isotropic free plate



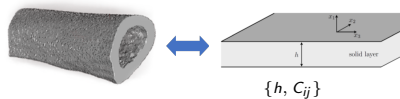
Hypothesis 2: homogenized elasticity



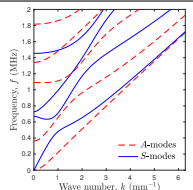
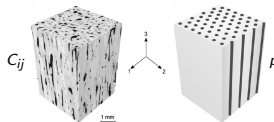
Waveguide models: solving the forward problem

1. Cortical bone assessment

Hypothesis 1: 2-D transverse isotropic free plate



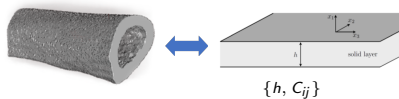
Hypothesis 2: homogenized elasticity



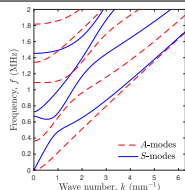
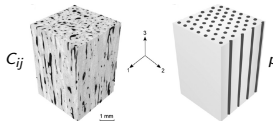
Waveguide models: solving the forward problem

1. Cortical bone assessment

Hypothesis 1: 2-D transverse isotropic free plate

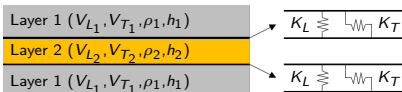
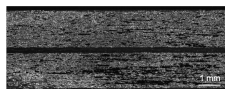


Hypothesis 2: homogenized elasticity



2. Adhesion of bonded layers

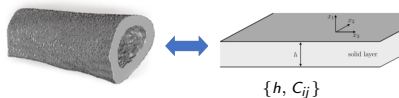
Hypothesis: rheological model for symmetric structures



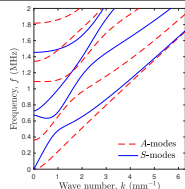
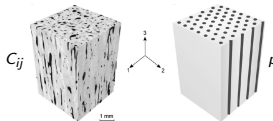
Waveguide models: solving the forward problem

1. Cortical bone assessment

Hypothesis 1: 2-D transverse isotropic free plate

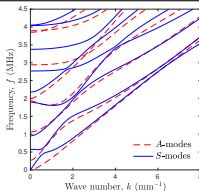
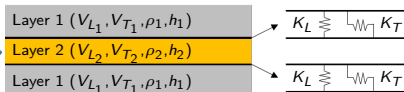
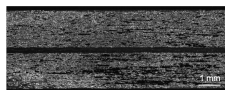


Hypothesis 2: homogenized elasticity



2. Adhesion of bonded layers

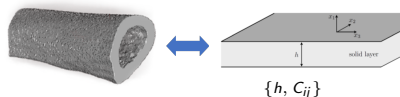
Hypothesis: rheological model for symmetric structures



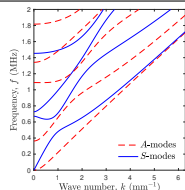
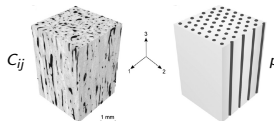
Waveguide models: solving the forward problem

1. Cortical bone assessment

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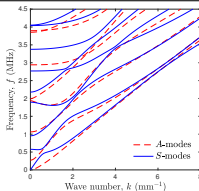
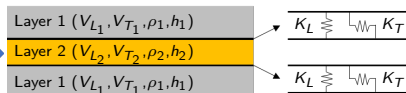
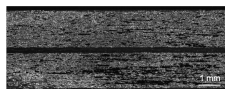


Hypothesis 2: homogenized elasticity



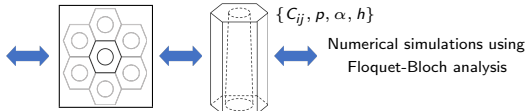
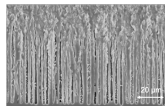
2. Adhesion of bonded layers

Hypothesis: rheological model for symmetric structures



3. Nanoporous silicon membranes

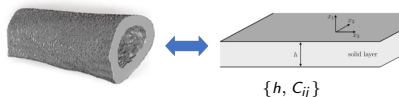
Hypothesis: conical-shaped mesopores distributed periodically



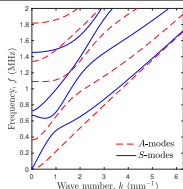
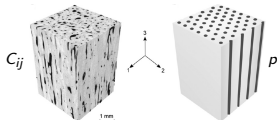
Waveguide models: solving the forward problem

1. Cortical bone assessment

Hypothesis 1: 2-D transverse isotropic free plate

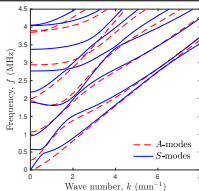
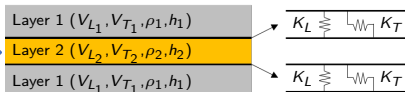
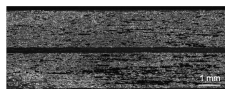


Hypothesis 2: homogenized elasticity



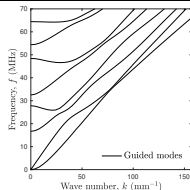
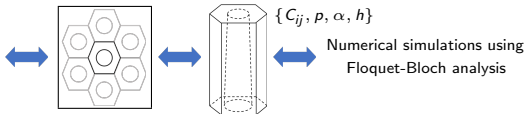
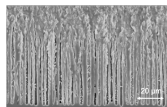
2. Adhesion of bonded layers

Hypothesis: rheological model for symmetric structures



3. Nanoporous silicon membranes

Hypothesis: conical-shaped mesopores distributed periodically

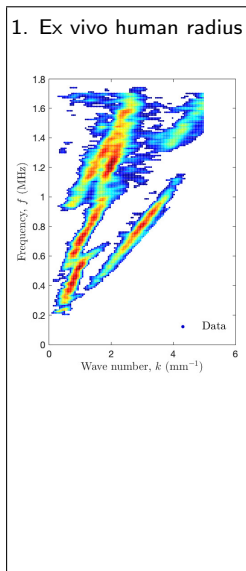


Multi-parametric inverse problem

Maximization of $F(\theta)$ using genetic algorithms: Example for a silicon wafer

Results: identification of the model parameters

1. Ex vivo human radius

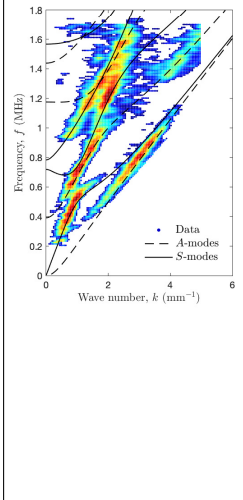


[1] Bochud et al, *Phys Med Biol*, 2016; [2] Bochud et al, *NDT&E Int*, 2019 (in prep.); [3] Thelen et al, 2019 (in prep.).

Results: identification of the model parameters

1. Ex vivo human radius

Estimates: $\hat{\theta} = [2.3 \text{ mm}; 6.0\%]$

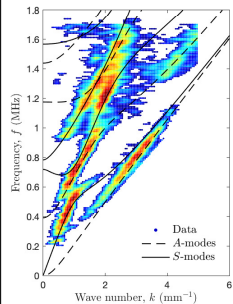


[1] Bochud et al, *Phys Med Biol*, 2016; [2] Bochud et al, *NDT&E Int*, 2019 (in prep.); [3] Thelen et al, 2019 (in prep.).

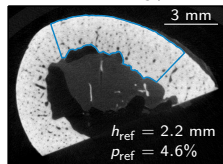
Results: identification of the model parameters

1. Ex vivo human radius

Estimates: $\hat{\theta} = [2.3 \text{ mm}; 6.0\%]$



Validation using μ -CT

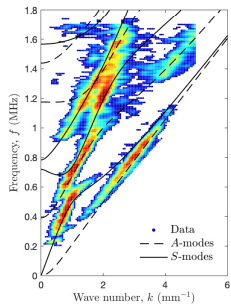


[1] Bochud et al, *Phys Med Biol*, 2016; [2] Bochud et al, *NDT&E Int*, 2019 (in prep.); [3] Thelen et al, 2019 (in prep.).

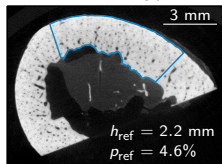
Results: identification of the model parameters

1. Ex vivo human radius

Estimates: $\hat{\theta} = [2.3 \text{ mm}; 6.0\%]$



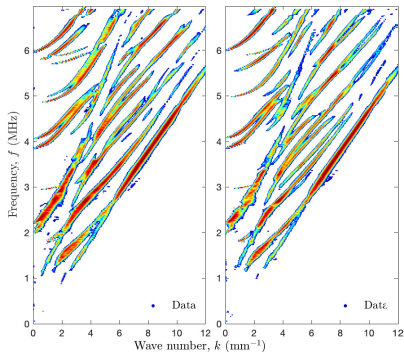
Validation using μ -CT



2. Adhesion of bonded layers

Good mechanical strength

Bad mechanical strength

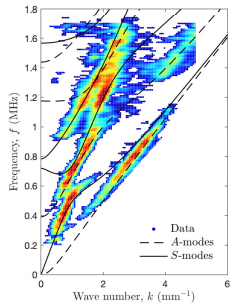


[1] Bochud et al, *Phys Med Biol*, 2016; [2] Bochud et al, *NDT&E Int*, 2019 (in prep.); [3] Thelen et al, 2019 (in prep.).

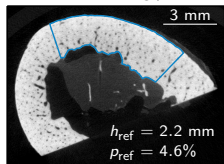
Results: identification of the model parameters

1. Ex vivo human radius

Estimates: $\hat{\theta} = [2.3 \text{ mm}; 6.0\%]$



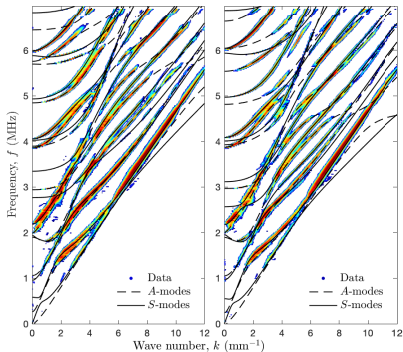
Validation using μ -CT



2. Adhesion of bonded layers

Good mechanical strength

Bad mechanical strength



Estimates: $\theta = [V_{L_2} \ V_{T_2} \ h_2 \ K_L \ K_T]$

$$\hat{\theta}_G = [2.5 \ 1.1 \ 180 \ 10^5 \ 10^4]$$

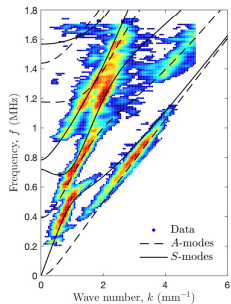
$$\hat{\theta}_B = [2.4 \ 1.0 \ 190 \ 10^4 \ 10^3]$$

[1] Bochud et al, *Phys Med Biol*, 2016; [2] Bochud et al, *NDT&E Int*, 2019 (in prep.); [3] Thelen et al, 2019 (in prep.).

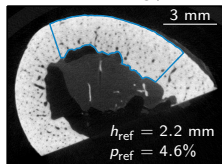
Results: identification of the model parameters

1. Ex vivo human radius

Estimates: $\hat{\theta} = [2.3 \text{ mm}; 6.0\%]$



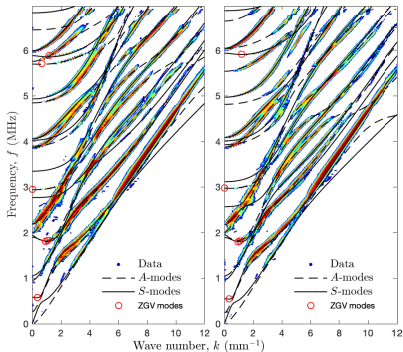
Validation using μ -CT



2. Adhesion of bonded layers

Good mechanical strength

Bad mechanical strength

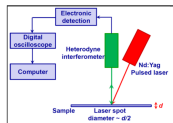


Estimates: $\theta = [V_{L2} \ V_{T2} \ h_2 \ K_L \ K_T]$

$$\hat{\theta}_G = [2.5 \ 1.1 \ 180 \ 10^5 \ 10^4]$$

$$\hat{\theta}_B = [2.4 \ 1.0 \ 190 \ 10^4 \ 10^3]$$

Validation using ZGV modes measured by laser ultrasonics

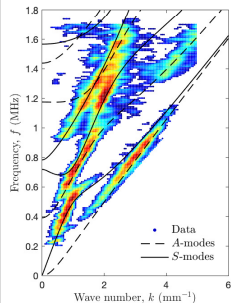


[1] Bochud et al, *Phys Med Biol*, 2016; [2] Bochud et al, *NDT&E Int*, 2019 (in prep.); [3] Thelen et al, 2019 (in prep.).

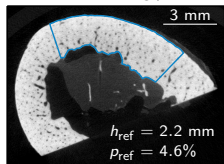
Results: identification of the model parameters

1. Ex vivo human radius

Estimates: $\hat{\theta} = [2.3 \text{ mm}; 6.0\%]$



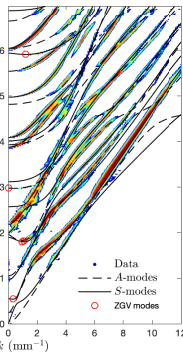
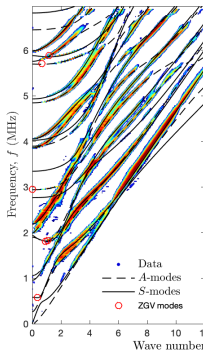
Validation using μ -CT



2. Adhesion of bonded layers

Good mechanical strength

Bad mechanical strength

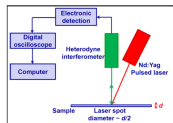


Estimates: $\theta = [V_{L2} \ V_{T2} \ h_2 \ K_L \ K_T]$

$$\hat{\theta}_G = [2.5 \ 1.1 \ 180 \ 10^5 \ 10^4]$$

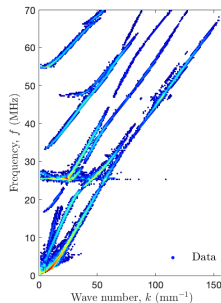
$$\hat{\theta}_B = [2.4 \ 1.0 \ 190 \ 10^4 \ 10^3]$$

Validation using ZGV modes measured by laser ultrasonics



3. Nanoporous silicon

npSi sample with 50% porosity

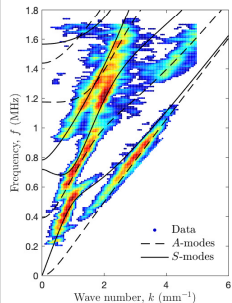


[1] Bochud et al, *Phys Med Biol*, 2016; [2] Bochud et al, *NDT&E Int*, 2019 (in prep.); [3] Thelen et al, 2019 (in prep.).

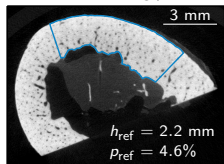
Results: identification of the model parameters

1. Ex vivo human radius

Estimates: $\hat{\theta} = [2.3 \text{ mm}; 6.0\%]$



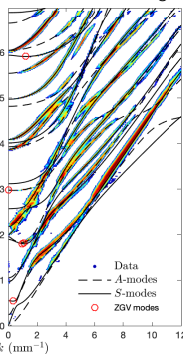
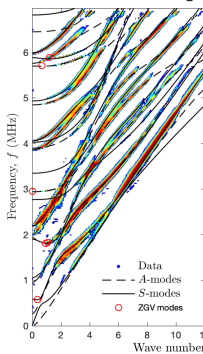
Validation using μ -CT



2. Adhesion of bonded layers

Good mechanical strength

Bad mechanical strength

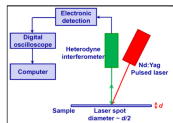


Estimates: $\theta = [V_{L_2} \ V_{T_2} \ h_2 \ K_L \ K_T]$

$\hat{\theta}_G = [2.5 \ 1.1 \ 180 \ 10^5 \ 10^4]$

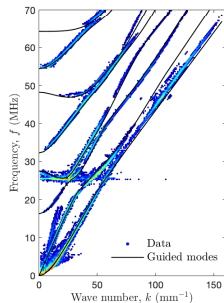
$\hat{\theta}_B = [2.4 \ 1.0 \ 190 \ 10^4 \ 10^3]$

Validation using ZGV modes measured by laser ultrasonics

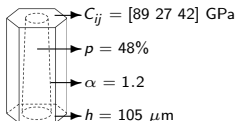


3. Nanoporous silicon

npSi sample with 50% porosity



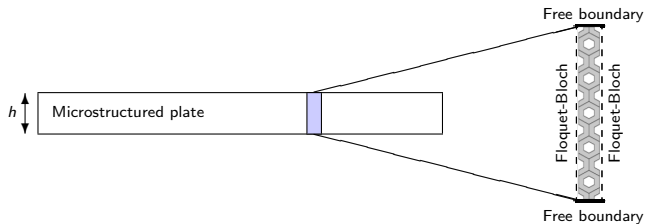
Estimates: $\theta = [C_{ij} \ p \ \alpha \ h]$



[1] Bochud et al, *Phys Med Biol*, 2016; [2] Bochud et al, *NDT&E Int*, 2019 (in prep.); [3] Thelen et al, 2019 (in prep.).

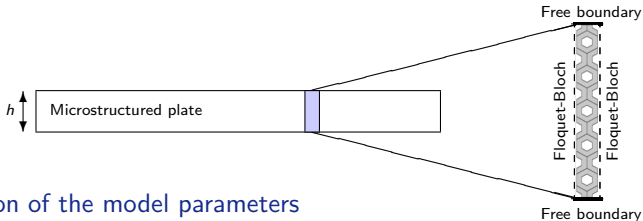
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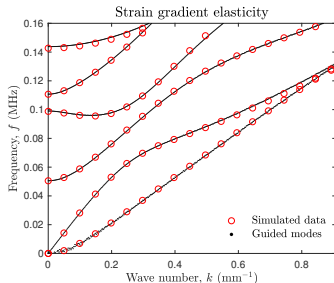
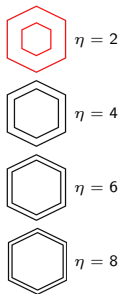
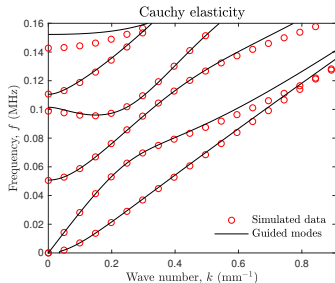


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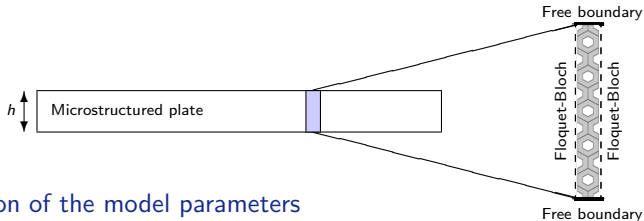


Identification of the model parameters

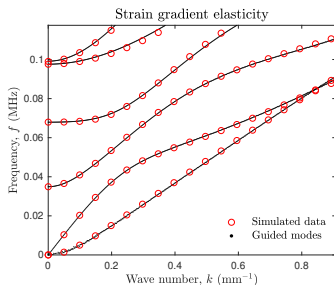
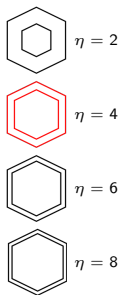
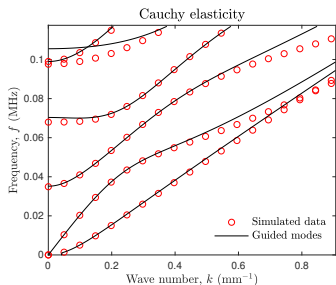


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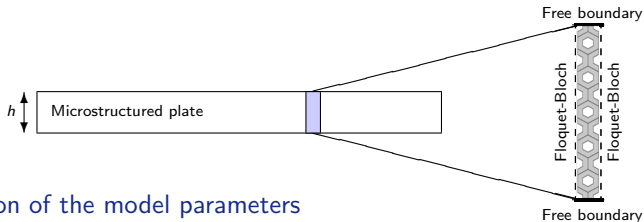


Identification of the model parameters

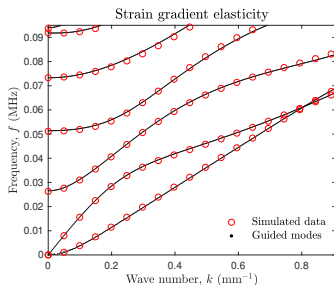
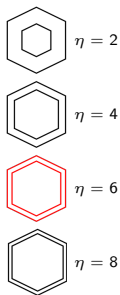
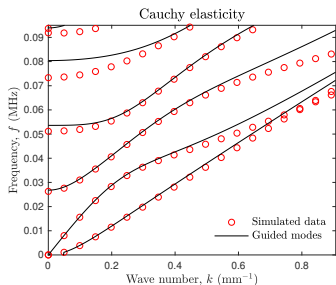


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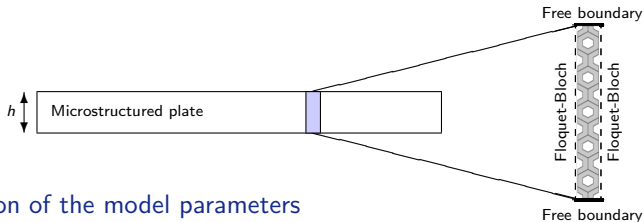
Identification of the model parameters



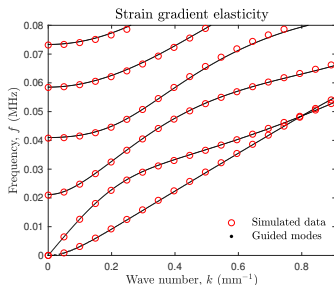
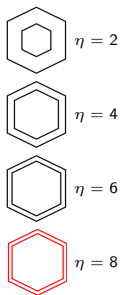
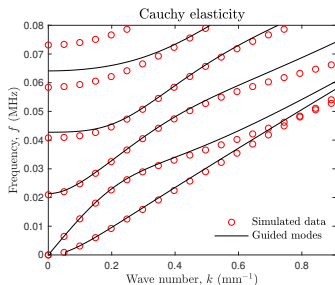
[1] Rosi et al, *Continuum Mech Therm*, 2019 (in prep.).

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Preliminary results on microstructured plates



Identification of the model parameters



Conclusion

- ▶ Two **experimental approaches** for measuring guided waves in complex media:
 - A **multi-element approach** using a programmable multi-channel electronics
 - + Real-time measurement ($f \in 0 - 15$ MHz) mm-thick samples
 - + Enhanced dispersion curves \Rightarrow SVD-based processing
 - Contact measurement \nrightarrow non-metallic or complex geometries
 - A non-contact **laser ultrasonics** approach:
 - + Extremely broadband measurement ($f \in 0 - 100$ MHz) μm -thick samples
 - + Dispersion curves and ZGV Lamb modes
 - Complex setup
- ▶ Associated **models** and **inverse problem** procedures to infer model parameters
 - Examples: cortical bone, bonded layers, nanoporous membranes
- ▶ Preliminary results on **microstructured** plates using strain gradient elasticity
 - Extend the study to 3D (e.g., gyroids) to mimic biological substitutes
 - Design microstructured samples using additive manufacturing
 - Measurements using ultrasound or vibro-acoustics

Thank you for your attention

