Coss&Vita

Invariant-based optimization methods for architectured structures

IRP Coss&Vita, M&MoCS, F2M

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October 17th - Kick-off meeting

COMECH

Introduction

Optimization of anisotropic standard structures

Optimization of architectured structures

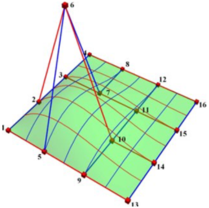
Computational mechanics of generalized continua (COMECH)

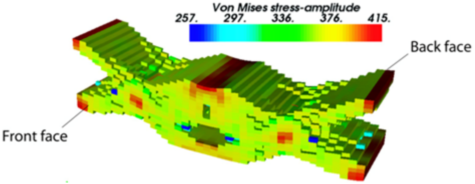
Main topics: (3 out of 4 are new)

- Numerical models for higher order continua based on Isogeometric interpolations
- Material and structural optimization algorithms
- Direct simulation of wave propagation in meta materials
- Numerical design and simulation of active elements composed by complex materials

Coordinators:

- **F2M:** Boris Desmorat
- **M&MoCS:** Massimo Cuomo, Leopoldo Greco





Past Actions (2015-2018)

- Workshops
  - Workshop on topic COMECH and NLS
    at Catania (Sicily), 29-31th of October 2015
  - Workshop Regularised models of brittle fracture
    at Université Pierre et Marie Curie (Paris), 2nd of May 2016
- Exchange of researcher
  - 2 weeks of Exchange were funded between France and Italy for the former COMECH research group

National contracts (ELADYN / COMECH)

- **ANR MoMaP** (2019-2023):
  Mesure et Optimisation des Matériaux Architecturés Périodiques
  Coordinator: M. Francois (GeM, Université de Nantes)
- **ANR Max-Oasis** (2020-2024):
  Matériaux Architecturés eXotiques, Ondes, AniSotropie, InStabilités
  Coordinator: N. Auffray (MSME, Marne-la-Vallée)

Computational mechanics of generalized continua (COMECH)

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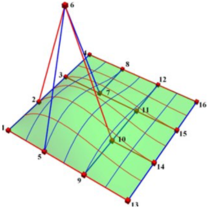
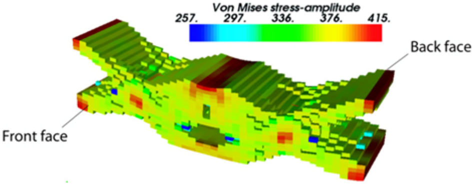




Study of Classical and generalized elasticity : physical motivations

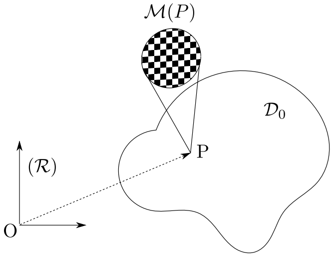
The constitutive law is an image of the microstructure of the material.

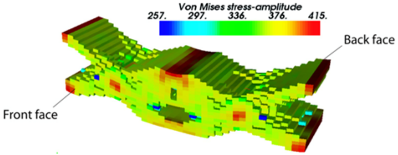
Connection with other topics

- **UP** : use of homogenization methods,. . .
- **NLS** : simulation of active elements, . . .
- **ELADYN**: Simulation of wave propagation, . . .

Optimal design

- Response to specifications;
- Achieving non standard properties;
- Controlling wave propagation;
- . . .





Optimal design using invariants

2D anisotropic Elasticity, small strains, small displacements

Structural problem

$\updownarrow$

Equivalent behavior
Invariant parametrization

$\updownarrow$

Micro structure level
Geometrical parametrization

Optimal design using invariants

2D anisotropic Elasticity, small strains, small displacements

Structural problem

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Equivalent behavior
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Micro structure level
Geometrical parametrization

Structural optimization

Objective function : Mass, Buckling, Energy
Optimization parameters : Invariants

Optimal design using invariants

2D anisotropic Elasticity, small strains, small displacements

Structural problem

$\updownarrow$

Equivalent behavior
Invariant parametrization

$\updownarrow$

Micro structure level
Geometrical parametrization

Structural optimization

Objective function : Mass, Buckling, Energy
Optimization parameters : Invariants

Optimal design of achitectured materials

Objective function : f(Invariants)
Optimization parameters : Geometry

Objectives of the talk

Aims of the talk

- introduce what is an invariant-based optimization method
- extend the approach to generalized continuum elasticity

The 2D setting

- complex enough to produce non trivial results
- simple enough to handle explicit computations
- construct situations that can be extended to the 3D problem.

COMECH

Introduction

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Optimization of architectured structures

THE ELASTICITY TENSOR

Hooke's law

Linear relation between the stress tensor $\sigma \in S^2(\mathbb{R}^2)$ and the strain tensor $\varepsilon \in S^2(\mathbb{R}^2)$:

$$\sigma = \mathbb{C} : \varepsilon$$

Properties

$\mathbb{C}$ is an element of the vector space $\mathbb{E}\mathrm{la} := S^2(S^2(\mathbb{R}^2))$;
$\mathbb{C}$ is positive definite :

$$\forall \varepsilon \neq 0, \quad \varepsilon : \mathbb{C} : \varepsilon > 0$$

An elastic material: a O(2) -orbit

O(2)-action

O(2) acts on $\mathbb{E}\mathrm{la}$ through standard $\star$ defined by :

$$\begin{aligned}\star : \mathrm{O}(2) \times \mathbb{E}\mathrm{la} &\to \mathbb{E}\mathrm{la}\\ (\mathrm{Q}, \mathbb{C}) &\mapsto \mathrm{Q} \star \mathbb{C} := Q_{ip}Q_{jq}Q_{kr}Q_{ls}C_{pqrs}\end{aligned}$$

Orbit

The set of tensors of $\mathbb{E}\mathrm{la}$ O(2)-conjugate to $\mathbb{C}$ constitutes its O(2)-orbit :

$$\mathrm{Orb}(\mathbb{C}) := \left\{ \overline{\mathbb{C}} = \mathrm{Q} \star \mathbb{C} \mid \mathrm{Q} \in \mathrm{O}(2) \right\}.$$

The orbits space is the quotient space $\mathbb{E}\mathrm{la}/\mathrm{O}(2)$.

Parametrization of 4th order elasticity tensor [Verchery 1979]

$$
\begin{aligned}
\mathrm{C}_{1111} &= T_0 + 2T_1 + R_0 \cos 4\Phi_0 + 4R_1 \cos 2\Phi_1 \\
\mathrm{C}_{1112} &= R_0 \sin 4\Phi_0 + 2R_1 \sin 2\Phi_1 \\
\mathrm{C}_{1122} &= -T_0 + 2T_1 - R_0 \cos 4\Phi_0 \\
\mathrm{C}_{1212} &= T_0 - R_0 \cos 4\Phi_0 \\
\mathrm{C}_{1222} &= -R_0 \sin 4\Phi_0 + 2R_1 \sin 2\Phi_1 \\
\mathrm{C}_{2222} &= T_0 + 2T_1 + R_0 \cos 4\Phi_0 - 4R_1 \cos 2\Phi_1
\end{aligned}
$$

$$
\begin{aligned}
T_0 &= \frac{1}{8}(\mathrm{C}_{1111} - 2\mathrm{C}_{1122} + 4\mathrm{C}_{1212} + \mathrm{C}_{2222}) \\
T_1 &= \frac{1}{8}(\mathrm{C}_{1111} + 2\mathrm{C}_{1122} + \mathrm{C}_{2222}) \\
R_0 e^{4i\Phi_0} &= \frac{1}{8}\left[\mathrm{C}_{1111} - 2\mathrm{C}_{1122} - 4\mathrm{C}_{1212} + \mathrm{C}_{2222} + 4i(\mathrm{C}_{1112} - \mathrm{C}_{1222})\right] \\
R_1 e^{2i\Phi_1} &= \frac{1}{8}\left[\mathrm{C}_{1111} - \mathrm{C}_{2222} + 2i(\mathrm{C}_{1112} + \mathrm{C}_{1222})\right]
\end{aligned}
$$

Transformation of a 4th order elasticity tensor

$$\mathrm{R}(\theta) : \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \qquad \mathrm{P}(\underline{\mathrm{e}}_1) : \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\mathbb{C} = (T_0, T_1, R_0 e^{4i\Phi_0}, R_1 e^{2i\Phi_1})$$

$$\mathrm{R}(\theta) \star \mathbb{C} = (T_0, T_1, R_0 e^{4i(\Phi_0+\theta)}, R_1 e^{2i(\Phi_1+\theta)})$$

$$\mathrm{P}(\underline{\mathrm{e}}_1) \star \mathbb{C} = (T_0, T_1, R_0 e^{4i\Phi_0}, R_1 e^{-2i\Phi_1})$$

2D elasticity tensor invariants

O(2)-integrity basis of $\mathbb{E}\mathrm{la}$

The following quantities

$$I_1 = T_1 \qquad J_1 = T_0 \qquad I_2 = R_1^2 \qquad J_2 = R_0^2 \qquad I_3 = R_0 R_1^2 \cos 4(\Phi_0 - \Phi_1)$$

- Constitute an integrity basis for the O(2)-action;
- The algebra $\mathbb{R}[\mathbb{E}\mathrm{la}]^{\mathrm{O}(2)}$ is free.

Bounds on the polar invariants

- The positive definiteness of $\mathbb{C}$ can be expressed in terms of bounds on its polar invariants
- It can be shown that the positive definiteness reduces to the following

$$\begin{aligned} & T_0 - |R_0| > 0, \\ & T_1(T_0^2 - R_0^2) - 2R_1^2\left[T_0 - R_0 \cos 4(\Phi_0 - \Phi_1)\right] > 0 \end{aligned}$$

- The above conditions $\Rightarrow T_0 > 0, T_1 > 0$.

Polar parameters of the inverse tensor

- The polar components of $\mathbb{S} = \mathbb{C}^{-1}$, denoted by the lower-case letters t_0, t_1, r_0, r_1 and $\varphi_0 - \varphi_1$, are:

$$
\begin{aligned}
t_0 &= \frac{2}{\Delta}\left(T_0 T_1 - R_1^2\right), \\
t_1 &= \frac{1}{2\Delta}\left(T_0^2 - R_0^2\right), \\
r_0 e^{4i\varphi_0} &= \frac{2}{\Delta}\left[(R_1 e^{2i\Phi_1})^2 - T_1 R_0 e^{4i\Phi_0}\right], \\
r_1 e^{2i\varphi_1} &= \frac{1}{\Delta}\left[R_0 e^{4i\Phi_0} R_1 e^{-2i\Phi_1} - T_0 R_1 e^{2i\Phi_1}\right].
\end{aligned}
$$

- Δ is an invariant positive quantity, defined by

$$\Delta = 8T_1\left(T_0^2 - R_0^2\right) - 16R_1^2\left[T_0 - R_0 \cos 4\left(\Phi_0 - \Phi_1\right)\right]$$

- An important result:

$$R_0 = 0 \nLeftrightarrow r_0 = 0.$$

R_0 SPECIAL-ORTHOTROPY

- $R_0 = 0$ identifies the so-called $R_0 - \textit{orthotropy}$
- With $R_0 = 0$, we get

$$
\begin{aligned}
C_{1111} &= \;\; T_0 + 2T_1 \quad + 4R_1 \cos 2\Phi_1 \\
C_{1112} &= \qquad\qquad\quad + 2R_1 \sin 2\Phi_1 \\
C_{1122} &= -T_0 + 2T_1 \\
C_{1212} &= \;\; T_0 \\
C_{1222} &= \qquad\qquad\quad + 2R_1 \sin 2\Phi_1 \\
C_{2222} &= \;\; T_0 + 2T_1 \quad - 4R_1 \cos 2\Phi_1
\end{aligned}
$$

- Two components are isotropic,
  the others rotates like those of a 2nd order tensor.

- Because $R_0 = 0 \not\Leftrightarrow r_0 = 0$, the dual case exists too: r_0-orthotropy.
- It concerns the compliance tensor $\mathbb{S}$. In such a case, it can be shown that

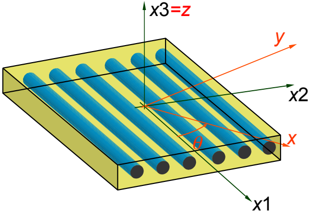
$$R_0 = \frac{R_1^2}{T_1}$$

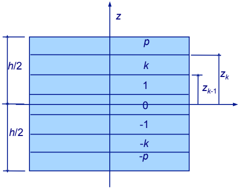
- The invariance of S_{1212} implies that of G_{12}:

$$G_{12} = \frac{1}{4S_{1212}} = \frac{1}{4t_0}.$$

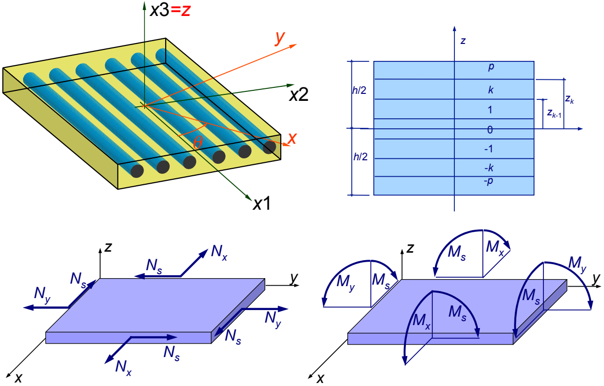
- This is the special orthotropy typical of paper (Vannucci, J Elas, 2010)

Abridged laminate mechanics





Abridged laminate mechanics



$$\left\{\frac{\mathbf{N}}{\mathbf{M}}\right\} = \left[\begin{array}{c|c} h\mathbb{A} & \frac{h^2}{2}\mathbb{B} \\ \hline \frac{h^2}{2}\mathbb{B} & \frac{h^3}{12}\mathbb{D} \end{array}\right]\left\{\frac{\varepsilon}{\chi}\right\} \quad \Leftrightarrow \quad \left\{\frac{\varepsilon}{\chi}\right\} = \left[\begin{array}{c|c} \frac{1}{h}\mathcal{A} & \frac{2}{h^2}\mathcal{B} \\ \hline \frac{2}{h^2}\mathcal{B}^\top & \frac{12}{h^3}\mathcal{D} \end{array}\right]\left\{\frac{\mathbf{N}}{\mathbf{M}}\right\}$$

The stiffness tensors

$$\mathbb{A} \to \begin{cases} T_0^A = T_0 \\ T_1^A = T_1 \\ R_0^A e^{4i\Phi_0^A} = R_0 e^{4i\Phi_0} (\xi_1 + i\xi_3) \\ R_1^A e^{2i\Phi_1^A} = R_1 e^{2i\Phi_1} (\xi_2 + i\xi_4) \end{cases}$$

$$\mathbb{B} \to \begin{cases} T_0^B = 0 \\ T_1^B = 0 \\ R_0^B e^{4i\Phi_0^B} = R_0 e^{4i\Phi_0} (\xi_5 + i\xi_7) \\ R_1^B e^{2i\Phi_1^B} = R_1 e^{2i\Phi_1} (\xi_6 + i\xi_8) \end{cases}$$

$$\mathbb{D} \to \begin{cases} T_0^D = T_0 \\ T_1^D = T_1 \\ R_0^D e^{4i\Phi_0^D} = R_0 e^{4i\Phi_0} (\xi_9 + i\xi_{11}) \\ R_1^D e^{2i\Phi_1^D} = R_1 e^{2i\Phi_1} (\xi_{10} + i\xi_{12}) \end{cases}$$

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Lamination parameters

$$\begin{cases} \xi_1 + i\xi_3 = \frac{1}{n}\sum_{j=1}^{n} e^{4i\delta_j} \\ \xi_2 + i\xi_4 = \frac{1}{n}\sum_{j=1}^{n} e^{2i\delta_j} \end{cases}$$

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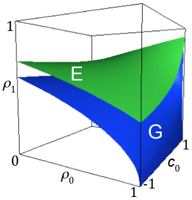
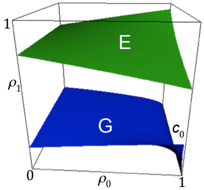
- Geometrical bounds for polar components of a laminate (Vannucci, J Elas, 2013)

$$\rho=\frac{R_0}{R_1},\quad \rho_0=\frac{R_0^{A,D}}{R_0},\quad \rho_1=\frac{R_1^{A,D}}{R_1},\quad \tau_0=\frac{T_0}{R_0},\quad \tau_1=\frac{T_1}{R_1}.$$

$$0\leq\rho_0,\quad 0\leq\rho_1,\quad \rho_0\leq 1,\quad 2\rho_1^2\leq\frac{1-\rho_0^2}{1-(-1)^{K^L}\rho_0\, c_0},\quad 2\rho_1^2<\rho\tau_0\tau_1\frac{1-\left(\frac{\rho_0}{\tau_0}\right)^2}{1-\frac{\rho_0}{\tau_0}\,c_0}.$$



Carbon-epoxy T-300/5208



Braided carbon-epoxy BR45-a

Microstructure examples

→ Orthotropic elementary layer

- Uncoupling
- Membrane : R_0-orthotropy
- Bending : R_0-orthotropy

$$[-11.1, 28.5, 25.2, -28.7, -24.5, 87.4, 40.0, 29.5, -24.1, 16.6, \\ 7.0, 49.2, 8.0, -14.9, 0.7, 56.0, -49.8, 37.9, -20.4, 11.5]$$

- Uncoupling
- Membrane : r_0-orthotropy
- Bending : r_0-orthotropy

$$[-17.6, -11.9, -36.4, 7.0, 25.9, -71.5, 59.8, -76.2, -14.1, 50.7, \\ -47.1, 16.9, 39.0, 86.1, -35.4, -70.1, -9.4, 25.0, -20.6, -15.0]$$

Design of anisotropic laminates

- Be: $\mathcal{P}=\{\mathcal{P}_i,\ i=1,...,12\}=\{R_0,R_1,\Phi_0-\Phi_1,\Phi_1\}_{A,B,D}$
  - $\mathbb{A}=\mathbb{A}(\mathcal{P}_i),\ \mathbb{B}=\mathbb{B}(\mathcal{P}_i),\ \mathbb{D}=\mathbb{D}(\mathcal{P}_i)$, unique correspondence

Design of anisotropic laminates

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  - functions $\mathcal{P}_i=\mathcal{P}_i(\delta_j)$ are not bijective

Design of anisotropic laminates

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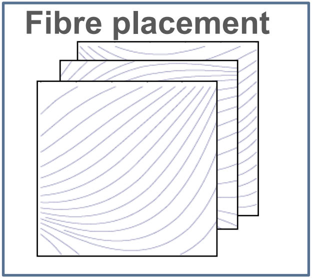
  - functions $\mathcal{P}_i=\mathcal{P}_i(\delta_j)$ are not bijective

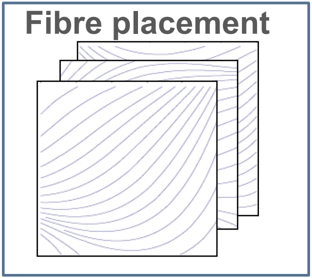
- Each problem is split into 2 subproblems, linked together and to be solved in sequence:
  - Step 1: the Structure Problem: design of the optimal anisotropy properties with respect to $f(x)$; the problem is formulated in the space of the $\mathcal{P}_i$s using the geometrical (feasibility) constraints
  - Step 2: the Constitutive Law Problem: determination of a suitable stacking sequence δ_j able to realize a laminate with the optimal $\mathcal{P}_i$s; non-bijectivity $\Rightarrow$ non-uniqueness.

Optimal anisotropic fields

- Idea: fibre placement
- Pb: properties (p. ex. $\mathbb{B} = 0$) are local
- Mathematically: optimization of three tensor fields of anisotropy, with local constraints



Optimal anisotropic fields



- Idea: fibre placement
- Pb: properties (p. ex. $\mathbb{B} = 0$) are local
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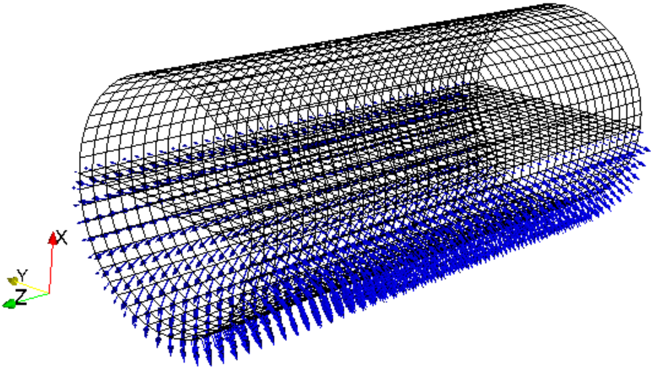
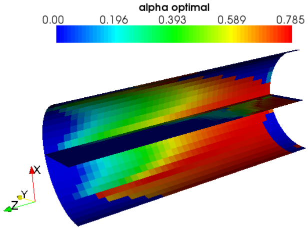
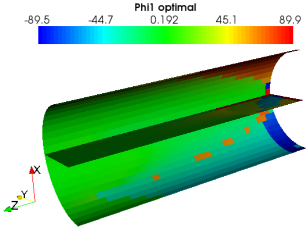
- Problems considered up till now:
  - stiffness optimization
    (PhD theses of C. Julien and A. Jibawy, 2010, Univ P6)
  - stiffness and strength optimization
    (PhD thesis of A. Catapano, 2013, Univ P6)

- Objective: minimization of the compliance; angle-ply laminates

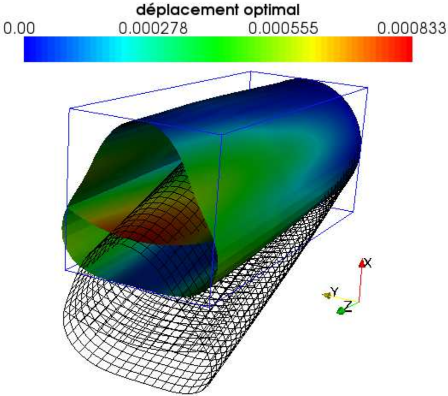




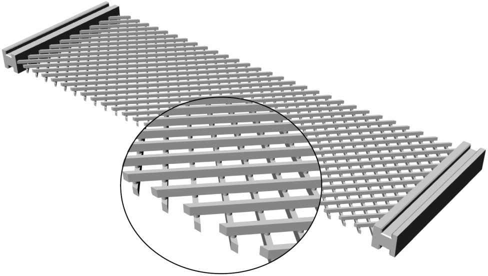
(a) Répartition de l'orientation optimale des fibres α^{opt}

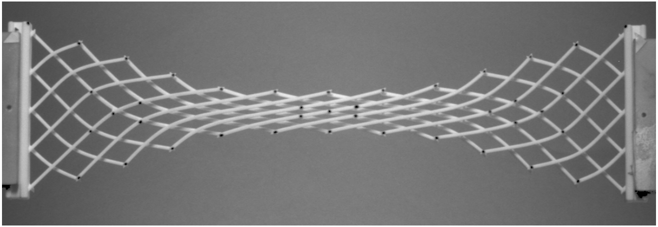


(b) Répartition des valeurs de Φ_1^{opt}



Pantograph - Bias test





Discrete model

- Stretching contribution of each micro-beam:

$$w_a = \frac{1}{2} k_a \left(\ell - \ell_0\right)^2$$

- Bending contribution of each micro-beam:

$$w_b = k_b \left(\cos\beta + 1\right)$$

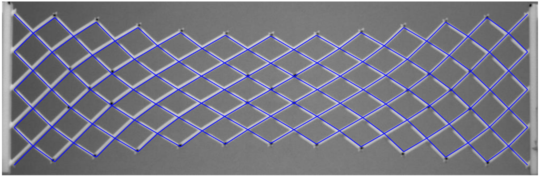
- Shear contribution of each pivot:

$$w_s = \frac{1}{2} \sum_{q=1}^{4} k_s \left(\gamma_q - \frac{\pi}{2}\right)^2$$

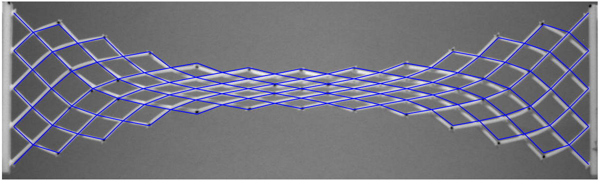
Discrete simulation

Pantograph dimensions: 70 mm x 210 mm

$k_a = 20$ N mm^{-1} $k_b = 20$ N mm $k_s = 2.7$ N mm

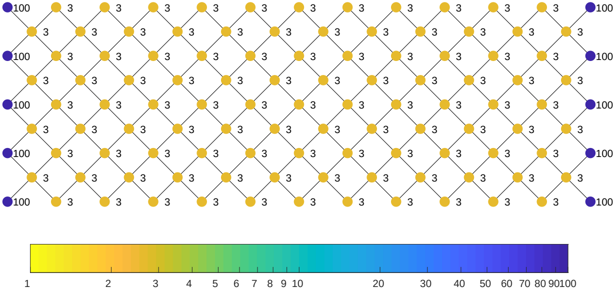


$28mm$

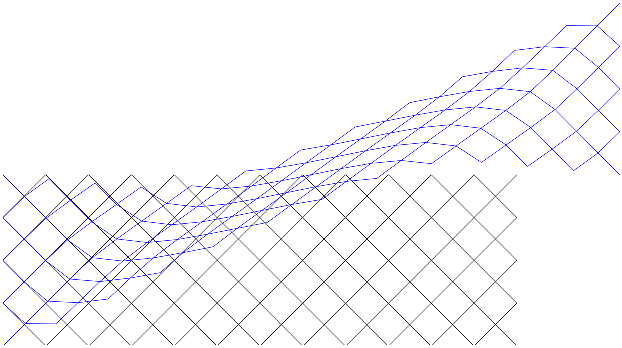


$56mm$

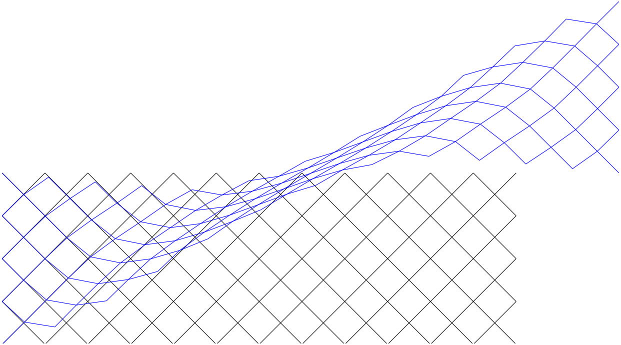
Shear-extension test: $\varepsilon_a^{max} = 5.2\%$, $R.u_d = 694$



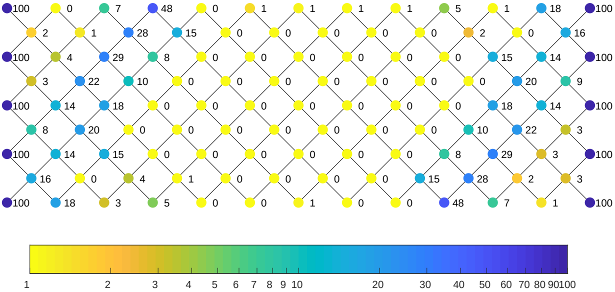
Shear-extension test: $\varepsilon_a^{max} = 5.2\%$, $R.u_d = 694$



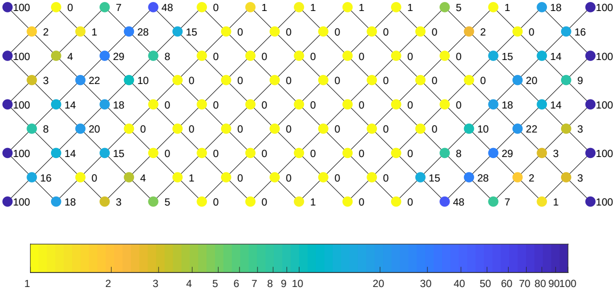
Shear-extension test: $\varepsilon_a^{max} = 4.3\%$, $R.u_d = 694$



Shear-extension test: $\varepsilon_a^{max} = 4.3\%$, $R.u_d = 694$



Shear-extension test: $\varepsilon_a^{max} = 4.3\%$, $R.u_d = 694$



→ Associated microstructure ?

Conclusion

Topology and anisotropy design of architectured structures

- Determine the optimal topology and field of anisotropy for a 2D generalized continuum with respect to a given objective function and constraints
- Find the associated microstructures