A 1D higher order gradient mixture model for porous media: the transitions from drained, undrained and unrelaxed regimes

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Outline Of The Talk

Definitions

1. Elastic moduls is defined, in the technical literature for saturated rocks and in the dynamic regime, as the ratio between the amplitude of the load and the amplitude of the deformation
2. For the elastic model such a ratio is independent of the frequency of the load
3. However, for experimental evidence shows a (paradoxical) dependence of the frequency of both elastic modulus and attenuation

The scheme of the experimental evidence

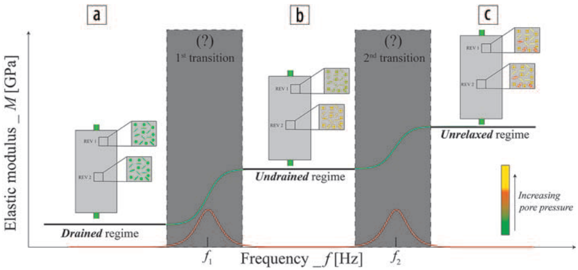


Figure: Elastic modulus and attenuation coefficient

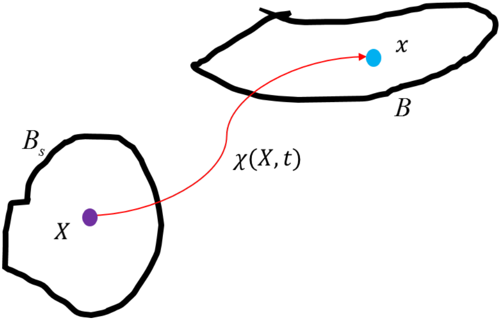
Consequences

1. The paradox of the dependence of the elastic modulus with respect to frequency is easily solved.
2. The elastic model is not the correct model to predict the results of these experiments.
3. The presence of two different dynamics suggests the use of mixture theory.
4. Static solutions will prove differences of drained, undrained and constrained-drained elastic moduli.
5. Dynamic solutions will prove dispersive behaviour.

Outline

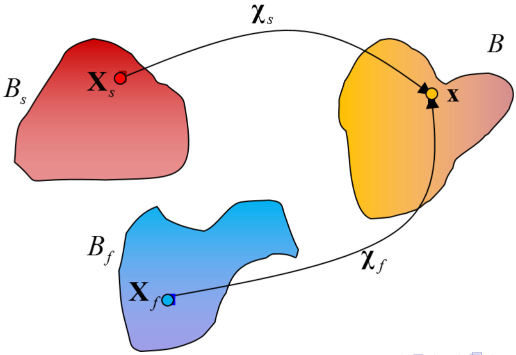
Classical continuum mechanics

One reference configuration $X \in B_s$
One present configuration $x \in B$
One placement χ such that $x = \chi(X, t)$



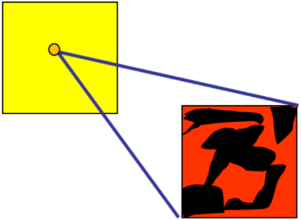
Mechanics of Mixtures

Two reference configurations $X_s \in B_s$ and $X_f \in B_f$
One present configuration $x \in B$
Two placements χ_s and χ_f such that $x = \chi_s(X_s, t) = \chi_f(X_f, t)$



Microstructures of Mixtures

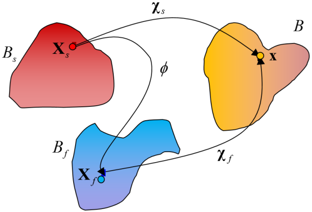
For each point $x \in B$ of the present configuration, a RVE is pretended to be described by the superposition of two points, $X_s \in B_s$ and $X_f \in B_f$, of two different reference configurations



The choice of the fundamental kinematical fields

The two placements $\chi_s(X_s, t)$ and $\chi_f(X_f, t)$ can not be chosen because their domains are different: $X_s \in B_s$ and $X_f \in B_f$

We define, at any time t, a function $\phi(X_s, t)$ that associates to each solid particle X_s that particular fluid material particle $X_f = \phi(X_s, t)$ occupying the same physical position $x = \chi_s(X_s, t) = \chi_f(X_f, t)$ as X_s.



Mass densities and porosity

The apparent mass densities of both species ρ_s and ρ_f, in the reference configuration, are defined by the respective masses M_s and M_f over the total volume V,

$$\rho_s = \frac{M_s}{V} = \frac{M_s}{V_s}\frac{V_s}{V} = \hat{\rho}_s \frac{V_s}{V}, \qquad \rho_f = \frac{M_f}{V_f}\frac{V_f}{V} = \hat{\rho}_f \frac{V_f}{V}. \tag{1}$$

Thus, they are related with the so-called true mass densities $\hat{\rho}_s = M_s/V_s$ and $\hat{\rho}_f = M_f/V_f$ through the volume fractions V_s/V and V_f/V or to the porosity$\nu = V_f/V$, that here will be used only for better characterize the values of the apparent mass densities ρ_s and ρ_f in the reference configuration.

Outline

The 1D case

In order to simplify the equations that follow, we consider the simple $1D$ case, i.e.,

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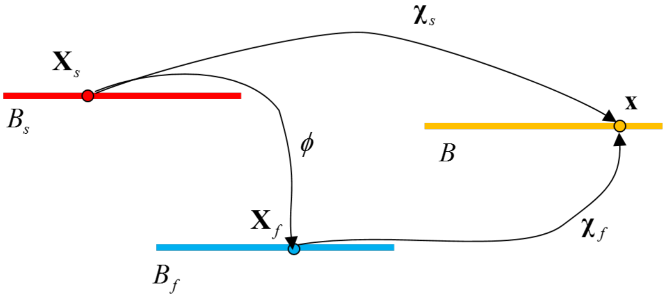

Displacement fields

We can define two displacement fields,

$$u_s(X_s,t)=\chi_s(X_s,t)-X_s, \qquad u_f(X_f,t)=\chi_f(X_f,t)-X_f,$$

one for the solid, the displacement u_s, and one for the fluid, the displacement u_f. The displacement relative to the second kinematical function $\phi(X_s,t)$,

$$X_f=\phi(X_s,t)=X_s+\varphi(X_s,t),$$

is therefore the function $\varphi(X_s,t)$ that gives, for each solid particle X_s, the displacement of the fluid particle X_f that was in contact with X_s.

Displacement fields

The relation between the three defined displacement fields is the following

$$u_f\left(X_f,t\right)=x-X_f=X_s+u_s-X_s-\varphi=u_s\left(X_s,t\right)-\varphi\left(X_s,t\right)$$

Outline

The extended Rayleigh-Hamilton principle

Formulation of the principle

$$\delta A = \int_{t_i}^{t_f} \frac{\partial R}{\partial \dot{q}_i} \delta q_i dt, \quad \forall \delta q / \delta q(t = t_i) = \delta q(t = t_i) = 0, \qquad (2)$$

where the kinematical fields q and the action A are defined.

$$q = \left\{ u_s, \varphi, u_s^{'}, \varphi^{'} \right\}, \quad A = \int_{t_i}^{t_f} \left(K - U + U^{ext} \right) dt.$$

The Kinetic energy

Definition of the kinetic energy

$$K = \int_0^L \left(\frac{1}{2}\rho_s \dot{u}_s^2 + \frac{1}{2}\rho_f \dot{u}_f^2 + \frac{1}{2}\eta_s \dot{u}_s'^2 + \frac{1}{2}\eta_f \left(\frac{\partial \dot{u}_f}{\partial X_f} \right)^2 \right) dX_s, \qquad (3)$$

where superimposed dot and apex mean, respectively, the derivative with respect to time t and to position X_s.
We remark that the first two terms in (3) are the standard kinetic energies for the two species of the mixture and give the standard inertial contributions.

The Kinetic energy

Definition of the kinetic energy

$$K = \int_0^L \left(\frac{1}{2}\rho_s \dot{u}_s^2 + \frac{1}{2}\rho_f \dot{u}_f^2 + \frac{1}{2}\eta_s \dot{u}_s'^2 + \frac{1}{2}\eta_f \left(\frac{\partial \dot{u}_f}{\partial X_f} \right)^2 \right) dX_s, \qquad (4)$$

Besides, the last two terms are the so-called micro-inertial terms and give the contribution of the microstructures to the inertia. Thus, η_s and η_f are the so-called micro-inertias of the two species. Notwithstanding it would be possible to add interaction terms for inertial and micro-inertial contributions, this is avoided.

The strain energy

The strain energy (i.e., the internal energy functional) is also defined,

$$U = \int_0^L \left(\frac{1}{2} \kappa_s u_s^{\prime 2} + \frac{1}{2} \kappa_f \left(\frac{\partial u_f}{\partial X_f} \right)^2 + \kappa_{sf} u_s^{\prime} \frac{\partial u_f}{\partial X_f} + \frac{1}{2} \kappa_{sm} u_s^{\prime\prime 2} + \frac{1}{2} \kappa_{fm} \left(\frac{\partial^2 u_f}{\partial X_f^2} \right) \tag{5}$$

where the axial stiffnesses of the two species κ_s and κ_f are introduced as well as a conservative interaction term (the third of the previous equation). The parameter κ_{sf} is called the conservative interaction stiffness.

The strain energy

The strain energy (i.e., the internal energy functional) is also defined,

$$U=\int_0^L\left(\frac{1}{2}\kappa_s u_s'^2+\frac{1}{2}\kappa_f\left(\frac{\partial u_f}{\partial X_f}\right)^2+\kappa_{sf}u_s'\frac{\partial u_f}{\partial X_f}+\frac{1}{2}\kappa_{sm}u_s''^2+\frac{1}{2}\kappa_{fm}\left(\frac{\partial^2 u_f}{\partial X_f^2}\right)^2\right. \tag{6}$$

Higher order gradient contributions to the internal energy are represented by the last two terms of (6). κ_{sm} and κ_{fm} are called the stiffnesses of the microstructures. Even in this case, it would be possible to add an interaction term that, for the same sake of simplicity, is avoided in the present model.

The external energy functional

In order to take into account the effects of the external word, the external energy functional U^{ext} is also defined

$$U^{ext} = \int_0^L \left[b_s^{ext} u_s + b_f^{ext} u_f \right] dX_s + \\ + \left[f_s^{ext} u_s + f_f^{ext} u_f + d_s^{ext} u_s' + d_f^{ext} \frac{\partial u_f}{\partial X_f} \right]_{X_s=0} + \\ + \left[f_s^{ext} u_s + f_f^{ext} u_f + d_s^{ext} u_s' + d_f^{ext} \frac{\partial u_f}{\partial X_f} \right]_{X_s=L},$$

where the integral part is due to distributed forces that are acted by the external word on the two species, i.e. b_s^{ext} and b_f^{ext}. For the same sake of simplicity, it is avoided the contributions of distributed external double forces.

Stress partitiong law

Concentrated forces f_s^{ext}, f_f^{ext} and double forces d_s^{ext}, d_f^{ext} on both species are possible to be prescribed at the boundaries, in $X_s = 0$ and/or in $X_s = L$, in this model in an INDEPENDENT way. It must be underlined that any prescription on the repartition of a certain external force f^{ext} on the two species of the mixture,i.e.,

$$f_s^{ext}\left(f^{ext}\right), f_f^{ext}\left(f^{ext}\right)$$

is a constitutive assumption that should be avoided for the general case. A remarkable example is the case where kinematical (i.e., displacement) and dual (i.e., force) conditions are assumed for the two species.

Rayleigh energy functional

The dissipation Rayleigh energy functional is also defined,

$$R = \int_0^L \left[\frac{1}{2} D \left(\dot{u}_s - \dot{u}_f \right)^2 + \frac{1}{2} D_m \left(\dot{u}_s' - \frac{\partial \dot{u}_f}{\partial X_f} \right)^2 \right] dx, \tag{7}$$

where D is the standard Darcy viscosity coefficient and D_m is an higher order Darcy term, due to the Brinkman dissipation.

Thermodynamic restrictions

Positive definiteness of the kinetic K, internal U and Rayleigh R energy functionals implies the following thermodynamically restrictions on the constitutive parameters of this model,

$$\rho_\alpha > 0, \quad \eta_\alpha > 0, \quad \kappa_\alpha > 0, \quad \kappa_{\alpha m} > 0, \tag{8}$$

$$D > 0, \quad D_m > 0, \quad |\kappa_{sf}| < \kappa_s \kappa_f, \quad \alpha = s, f. \tag{9}$$

Outline

Technical advices

In order to derive the system of PDEs and boundary conditions, first of all the kinetic, the strain and the external energies must be expressed in terms of the two fundamental kinematical fields

$$u_s\left(X_s,t\right),\varphi\left(X_s,t\right). \tag{10}$$

Then, the variation of the three functional must be calculated and the principle (2) must be imposed for any kinematical fields (10) that satisfy that kinematical boundary conditions.

Partial differential equations

The two partial differential equations are derived

$$\rho_f \ddot{\varphi} - (\rho_s + \rho_f)\,\ddot{u}_s + F_u^{'} + \left(b_s^{ext} + b_f^{ext}\right) = 0 \qquad (11)$$

$$+\rho_f \ddot{\varphi} - \rho_f \ddot{u}_s + F_\varphi^{'} + b_f^{ext} + D\dot{\varphi} - D_m \dot{\varphi}^{''} = 0 \qquad (12)$$

where

$$M_u = (\kappa_{sm} + \kappa_{fm})\,u_s^{''} - \kappa_{fm}\varphi^{''}$$

$$M_\varphi = -\kappa_{fm}\varphi^{''} + \kappa_{fm}$$

$$F_u = (\kappa_s + \kappa_f + 2\kappa_{sf})\,u_s^{'} + (-\kappa_f - \kappa_{sf})\,\varphi^{'} - (\kappa_{sm} + \kappa_{fm})\,u_s^{'''}$$
$$+\kappa_{fm}\varphi^{'''} + (\eta_s + \eta_f)\,\ddot{u}_s^{'} - \eta_f \ddot{\varphi}^{'}$$

$$F_\varphi = (\kappa_f + \kappa_{sf})\,u_s^{'} - \kappa_f \varphi^{'} + \kappa_{fm}\varphi^{'''} - \kappa_{fm} u_s^{'''} +$$
$$-\eta_f \ddot{\varphi}^{'} + \eta_f \ddot{u}_s^{'} - D_m \dot{\varphi}^{'}$$

Boundary Conditions

We have the following system of Boundary conditions,

$$X_s = L \qquad + \left[F_u - \left(f_s^{ext} + f_f^{ext}\right)\right] \delta u_s \tag{13}$$

$$X_s = 0 \qquad + \left[F_u + \left(f_s^{ext} + f_f^{ext}\right)\right] \delta u_s \tag{14}$$

$$X_s = L \qquad \left[F_\varphi - f_f^{ext}\right] \delta \varphi \tag{15}$$

$$X_s = 0 \qquad \left[F_\varphi + f_f^{ext}\right] \delta \varphi \tag{16}$$

$$X_s = L \qquad \left[M_u - \left(d_s^{ext} + d_f^{ext}\right)\right] \delta u_s^{'} \tag{17}$$

$$X_s = 0 \qquad \left[M_u + \left(d_s^{ext} + d_f^{ext}\right)\right] \delta u_s^{'} \tag{18}$$

$$X_s = L \qquad \left[M_\varphi - d_f^{ext}\right] \delta \varphi^{'} \tag{19}$$

$$X_s = 0 \qquad \left[M_\varphi + d_f^{ext}\right] \delta \varphi^{'} \tag{20}$$

Outline

Boundary conditions for the undrained problem

Assumptions

$$\eta_f = \kappa_{fm} = b_s^{ext} = b_f^{ext} = 0$$

Assumed kinematic boundary conditions:

$$u_s\left(X_s = 0\right) = 0 \tag{21}$$
$$\varphi\left(X_s = 0\right) = 0 \tag{22}$$
$$\varphi\left(X_s = L\right) = 0 \tag{23}$$

Assumed dynamic boundary conditions:

$$d_s^{ext} + d_f^{ext} = 0 \qquad X_s = 0 \tag{24}$$
$$f_s^{ext} + f_f^{ext} = F^{ext} \qquad X_s = L \tag{25}$$
$$d_s^{ext} + d_f^{ext} = 0 \qquad X_s = L \tag{26}$$

Solution for the undrained problem

The solution is unique and it is

$$u_s(X_s) = F^{ext} \frac{X_s}{\kappa_s + \kappa_f + 2\kappa_{sf}}, \qquad \varphi(x) = 0,$$

we can deduce the interpretation of the undrained compressibility $K_s^{undr} = F^{ext} / u_s^{'}$

$$K_s^{undr} = \kappa_s + \kappa_f + 2\kappa_{sf}, \tag{27}$$

Outline

Boundary conditions for the free drained problem

Assumptions

$$\eta_f = \kappa_{fm} = b_s^{ext} = b_f^{ext} = 0$$

Assumed kinematic boundary conditions:

$$u_s\left(X_s = 0\right) = 0 \tag{28}$$

$$\varphi\left(X_s = 0\right) = 0 \tag{29}$$

Assumed dynamic boundary conditions:

$$d_s^{ext} + d_f^{ext} = 0 \qquad X_s = 0 \tag{30}$$

$$f_s^{ext} + f_f^{ext} = F^{ext} \qquad X_s = L \tag{31}$$

$$f_f^{ext} = 0 \qquad X_s = L \tag{32}$$

$$d_s^{ext} + d_f^{ext} = 0 \qquad X_s = L \tag{33}$$

Solution for the free drained problem

The solution is again unique and given by the following expressions of the displacement fields,

$$u_s(X_s) = F^{ext} X_s \frac{\kappa_f}{\kappa_s \kappa_f - \kappa_{sf}^2}, \qquad \varphi(x) = F^{ext} X_s \frac{\kappa_f + \kappa_{sf}}{\kappa_s \kappa_f - \kappa_{sf}^2}, \quad (34)$$

where we can deduce another interpretation of the solid compressibility in the drained condition $K_s^{dr} = F^{ext}/u_s^{'}$,

$$K_s^{dr} = \frac{\kappa_s \kappa_f - \kappa_{sf}^2}{\kappa_f} = \kappa_s - \frac{\kappa_{sf}^2}{\kappa_f}, \quad (35)$$

Because of the thermodynamic restrictions we have that

$$K_s^{undr} \geq K_s^{dr}$$

that the solid displacement has the same sign of the external force F^{ext}.

The hydrophobic and hydrophilic conditions

We have finally that the fluid displacement,

$$u_f = u_s - \varphi = F^{ext} X_s \frac{-\kappa_{sf}}{\kappa_s \kappa_f - \kappa_{sf}^2}$$

has the same sign of the external force F^{ext}, and therefore of the solid, only if

$$\kappa_{sf} < 0,$$

that gives a condition for the hydrophylic behaviour of the mixture. Besides, the condition

$$\kappa_{sf} < -\kappa_f$$

would mean a fluid displacement higher than the solid one.

Outline

Boundary conditions for the constrained and drained problem

Assumed kinematic boundary conditions:

$$u_s\left(X_s=0\right)=0 \tag{36}$$

$$\varphi\left(X_s=0\right)=0 \tag{37}$$

Assumed dynamic boundary conditions:

$$d_s^{ext}+d_f^{ext}=0 \qquad X_s=0 \tag{38}$$

$$f_s^{ext}+f_f^{ext}=F^{ext} \qquad X_s=L \tag{39}$$

$$f_f^{ext}=-p_f^{ext} \qquad X_s=L \tag{40}$$

$$d_s^{ext}+d_f^{ext}=0 \qquad X_s=L \tag{41}$$

where p_f^{ext} is a pressure applied to the fluid.

Solution for the constrained and drained problem

The solution is again unique and given by the following expressions of the displacement fields,

$$u_s(X_s) = X_s \frac{F^{ext}\kappa_f + p_f^{ext}(\kappa_f + \kappa_{sf})}{\kappa_s\kappa_f - \kappa_{sf}^2},$$

$$\varphi(X_s) = X_s \frac{F^{ext}(\kappa_f + \kappa_{sf}) + p_f^{ext}(\kappa_s + \kappa_f + 2\kappa_{sf})}{\kappa_s\kappa_f - \kappa_{sf}^2},$$

where we can deduce another interpretation of the solid compressibility $K_s = F/u_s^{'}$,

$$K_s = \frac{\kappa_s\kappa_f - \kappa_{sf}^2}{\kappa_f + \frac{p_f^{ext}}{F^{ext}}(\kappa_f + \kappa_{sf})}, \tag{42}$$

that gives the contribution of the fluid pressure on the solid compressibility.

Outline

Plane wave solution

Let us find a solution of the Partial differential equations in the following plane wave form,

$$u_s = Re\left\{u_s^0 \exp\left[I\left(\omega t - kX_s\right)\right]\right\}, \quad u_f = Re\left\{u_f^0 \exp\left[I\left(\omega t - kX_s\right)\right]\right\}, \tag{43}$$

where u_s^0 and u_f^0 are the complex wave amplitudes, ω is the frequency and k is the wave number. By insertion of (43) into the partial differential equations (11) and (12) we have a system of two algabraic equations into two unknowns.

Dispersion relation

The two algebraic equations has a non trivial solution in terms of the complex amplitudes only if the following dispersion relation is satisfied,

$$\left(\rho_s\omega^2 - \kappa_s k^2 - \kappa_{sm} k^4 + \eta_s \omega^2 k^2 - ID\omega + ID_m\omega k^2\right)\cdot$$

$$\cdot\left(\rho_f\omega^2 - \kappa_f k^2 - \kappa_{fm} k^4 + \eta_f \omega^2 k^2 - ID\omega + ID_m\omega k^2\right) = \tag{44}$$

$$= \left(-\kappa_{sf} k^2 + ID\omega - ID_m\omega k^2\right)^2 \tag{45}$$

Solution of the dispersion relation

The previous dispersion relation (45) can be solved as follows. Let us assume a real value for the frequency ω. In this case, the dispersion relation (45) is a polynomial of fourth order in the squared wave number variable k^2, that is possible to solve analytically. However, the results are very complicated and it is not possible to make that evident in a slide. Moreover, from such a complicated form one can not catch any interesting information. For this reason, it is better to show them in different ways. In the next slides we will analyze the low and high frequency regime. In the subsequent slides we plot the dispersion relation (45) in terms of phase velocity and attenuation coefficients.

Low frequency regime

Let us find a solution of (45) in the case of wave number and frequency have the same order of magnitude and in the low frequency regime. In this case the (45) is evaluated at the lowest (3^{rd}) order,

$$\frac{\omega}{k} = \sqrt{\frac{\kappa_s + \kappa_f + 2\kappa_{sf}}{\rho_s + \rho_f}} = \sqrt{\frac{K_s^{undr}}{\rho}} \tag{46}$$

Higher frequency regime

The same dispersion relation (45) at the higher (4^{th}) order, in the limit of

$$DD_m \ll \kappa_f \rho_s + \kappa_s \rho_f$$

we have that one analytical solution is

$$\frac{\omega}{k} = \sqrt{\frac{1}{2}\left(\frac{\kappa_s}{\rho_s} + \frac{\kappa_f}{\rho_f}\right) + \sqrt{4\left(\frac{\kappa_s}{\rho_s} - \frac{\kappa_f}{\rho_f}\right)^2 + \frac{\kappa_{sf}^2}{\rho_s \rho_f}}} \neq \sqrt{\frac{K_s^{dr}}{\rho}} \qquad (47)$$

Highest frequency regime

Finally, at the highest (7^{th}) order the (45) is evaluated and solved as follows,

$$\frac{\omega}{k} = \sqrt{\frac{\kappa_{sm}}{\eta_s}}. \tag{48}$$

We remark that at the highest frequency regime only second gradient coefficients and microinertia have a role

Outline

Numerical example

Let us call V^{ph}_{low} the phase velocity in the low frequency regime, V^{ph}_{med} the phase velocity in the intermediated plateau of the phase velocity and V^{ph}_{high} that in the highest frequency regime. From (47), (48) and (46) we have

$$V^{ph}_{low} = \sqrt{\frac{K^{undr}_s}{\rho}}, \qquad V^{ph}_{med} = \sqrt{\frac{1}{2}\left(\frac{\kappa_s}{\rho_s} + \frac{\kappa_f}{\rho_f}\right) + \sqrt{4\left(\frac{\kappa_s}{\rho_s} - \frac{\kappa_f}{\rho_f}\right)^2 + \frac{\kappa^2_{sf}}{\rho_s \rho_f}}}, \tag{49}$$

$$V^{ph}_{low} = 2900\,\mathrm{m/s}, \qquad V^{ph}_{med} = 3100\,\mathrm{m/s},$$

$$V^{ph}_{high} = 4000\,\mathrm{m/s},$$

Numerical assumptions on the apparent mass densities

We assume to have a sample of cylindrical shape with diameter d and height L. The porosity of the solid-fluid mixture is $\nu = 0.1$ and the three dimensional true mass density of the solid species is$\rho_s^{3D} = 2700\mathrm{Kg}/\mathrm{m}^3$. Thus, the apparent solid species mass density is,

$$\rho_s = \frac{d^2}{4}\pi\rho_s^{3D}(1-\nu) = 0.763\ \mathrm{Kg}/\mathrm{m}.$$

The three dimensional true mass density of the fluid species is$\rho_f^{3D} = 1000\mathrm{Kg}/\mathrm{m}^3$. Thus, the apparent fluid species mass density is,

$$\rho_f = \frac{d^2}{4}\pi\rho_f^{3D}\nu = 0.031\ \mathrm{Kg}/\mathrm{m}.$$

Numerical assumptions

Let us assume a $3D$ static solid compressibility $K_s^{3D} = 20\mathrm{GPa}$. Thus, if we have $K_s = K_s^{3D}\pi d^2/4 = 6.28\mathrm{MN}$ and the identification in the form (35), we have the following identification from (35) and from the first two of (49),

$$\kappa_s = 7.09\mathrm{MN}, \qquad \kappa_f = 70.9\mathrm{kN}, \qquad \kappa_{sf} = -238\mathrm{kN}.$$

In the highest frequency regime we can also set the following couple of values,

$$\eta_s = 1\,\mathrm{Kg\,m}, \qquad \kappa_{sm} = 16\mathrm{MNm}^2 = 16\mathrm{Kg\,m}^3\,\mathrm{s}^{-2}.$$

The Darcy coefficient D and the higher order Darcy coefficient D_m are identified a posteriori with the two picks in the attenuation figure, the second of Fig. ?? and set as follows,

$$D = 1\,\mathrm{Kg/m}, \qquad D_m = 200\,\mathrm{Kg\,m}.$$

Plots of the dispersion relation

We have already pointed out that the frequency ω is real. In this hypothesis the phase velocity V_{ph} and the attenuation coefficient Q^{-1} are defined as follows,

$$V_{ph} = Re\left(\frac{\omega}{k}\right) = \omega Re\left(\frac{1}{k}\right), \qquad Q^{-1} = 2\frac{Im(k)}{Re(k)},$$

and plotted as follows.

Plot of the phase velocity

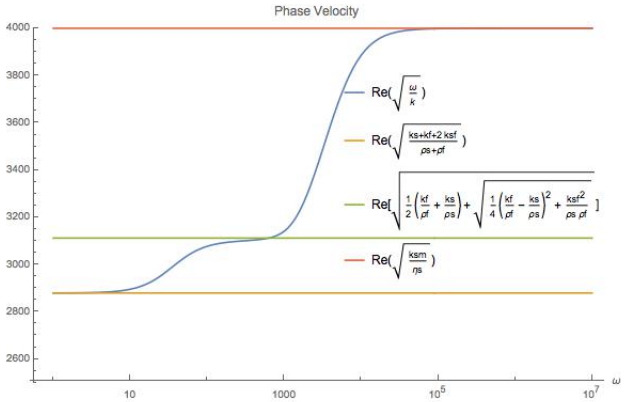

Figure: Phase velocity

Plot of the attenuation coefficient

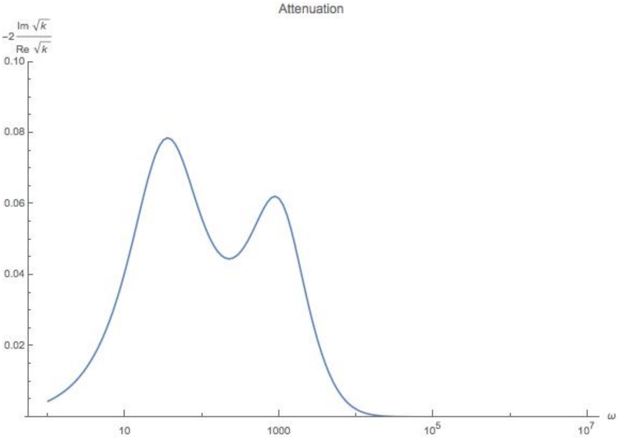

Figure: Attenuation coefficient

Conclusions

1. A 1D model for a saturated solid-fluid mixture is derived with the inclusion of higher order terms and through a variational procedure.
2. The model is conceived to get a well-posed system of PDEs and BCs that are able to take into account the transitions from drained, undrained an unrelaxed regimes.
3. Static analytical solutions are obtained. We derive differences of the elastic moduli for the drained, undrained and constrained-drained cases
4. Dynamic solutions are obtained in terms of plane waves.
5. In particular the dispersion relation of the mixture is achieved.
6. The diagrams of velocity and of the attenuation are shown

Conclusions

7. The first transition (at lower frequencies) is obtained because of the conceived two dynamics of the mixture.
8. The second transition (at higher frequencies) is obtained because of the inclusion of higher order term.
9. Nevertheless the simplicity of the model, they agree well with the experimental evidence.
Thank You For Your Attention