Upscaling methods for microstructured media (UP)

Arthur Lebée[1], Pierre Seppecher[2]

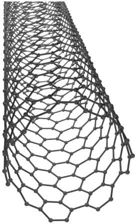
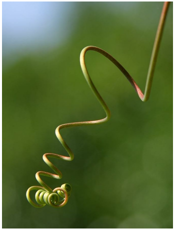
1: Laboratoire Navier (UMR CNRS 8205)
École des Ponts ParisTech; IFSTTAR; CNRS; Université Paris-Est

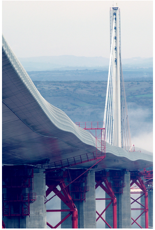
2: Laboratoire Imath
Université de Toulon

17/10/2019

Some illustrations of geometric regularity...

1D,

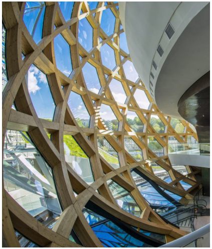



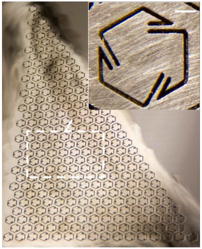


Geometric variable separation

Some illustrations of geometric regularity...

1D, 2D,

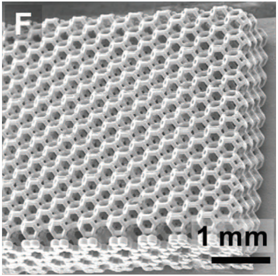


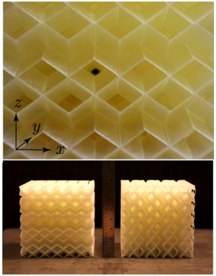


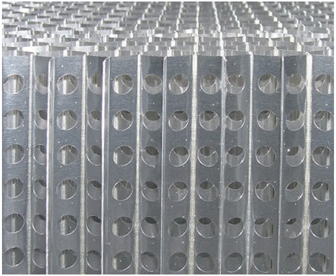
Geometric variable separation

Some illustrations of geometric regularity...

1D, 2D, 3D,

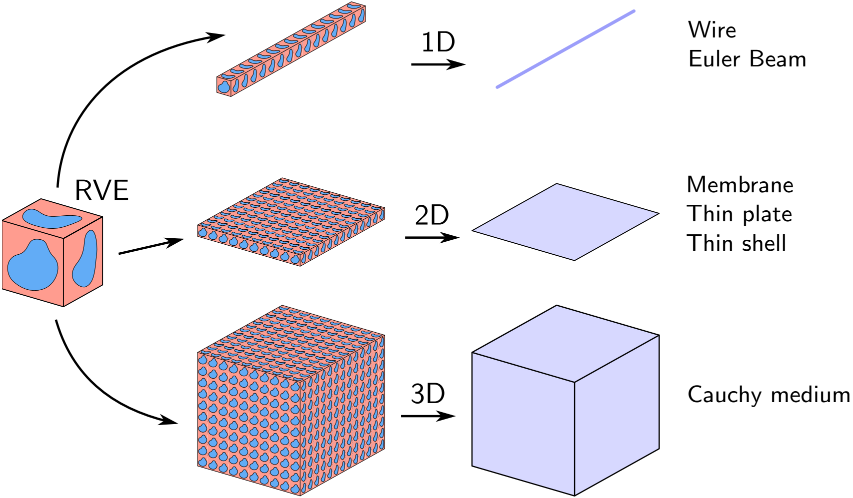




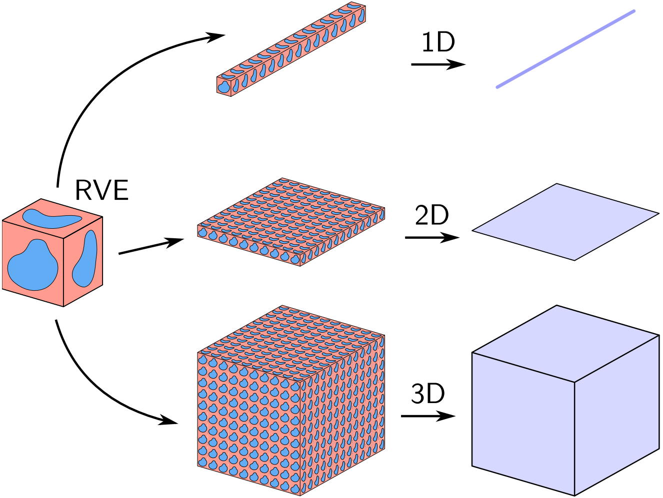


Geometric variable separation

Some equivalent elastic continua



Some equivalent elastic continua



Wire
Euler Beam
Timoshenko Beam
Pantographic Beam

Membrane
Thin plate
Thin shell
Thick plate
Thick shell

Cauchy medium
Strain gradient medium
Micromorphic medium

Upscaling methods for microstructured media (UP)

Derive predictive generalized continua from up-scaling methods?

- Tools
  - Asymptotic analysis and homogenization
  - Virtual/experimental testing:
    - Prototyping
    - Testing of non-standard properties?
- Applications
  - Architectured materials / Meta-materials
  - Smart materials
  - Modeling of slender structures

Continuum elasticity of Miura Tessellations

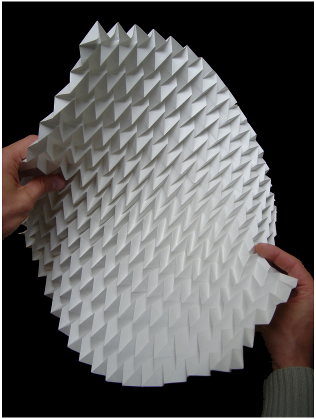
Hussein Nassar[1], Arthur Lebée[2], Laurent Monasse[3]

1: Dept. of Mechanical and Aerospace Engineering
University of Missouri, Columbia, USA

2: Laboratoire Navier (UMR CNRS 8205)
École des Ponts ParisTech; IFSTTAR; CNRS; Université Paris-Est, France

3: INRIA, Team Coffee and Laboratoire J.A. Dieudonné
Université Côte d'Azur, Nice, France

(Meta)-Surfaces from folded tessellations



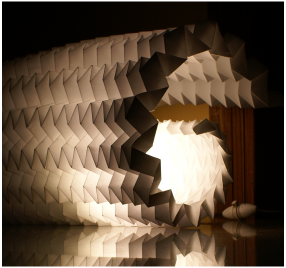
Resch and Christiansen (1970)

→ what are the accessible shapes?

→ what are the internal forces?

Free-form architecture from repetitive elements?

Most approaches are based on variations of patterns:



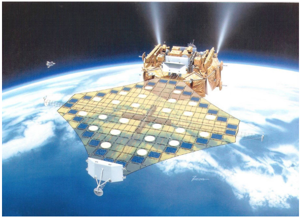
Lebée (2015)

Dudte et al. (2016)

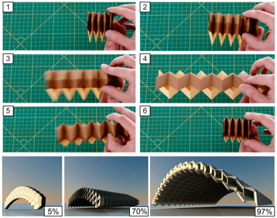
$\rightarrow$ True periodicity?

Large deformations of a (micro)-structured surface?

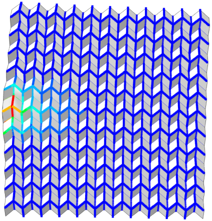
Deployable structures?/Morphing shells?/Meta-materials?



Miura (1993)



Filipov et al. (2015)

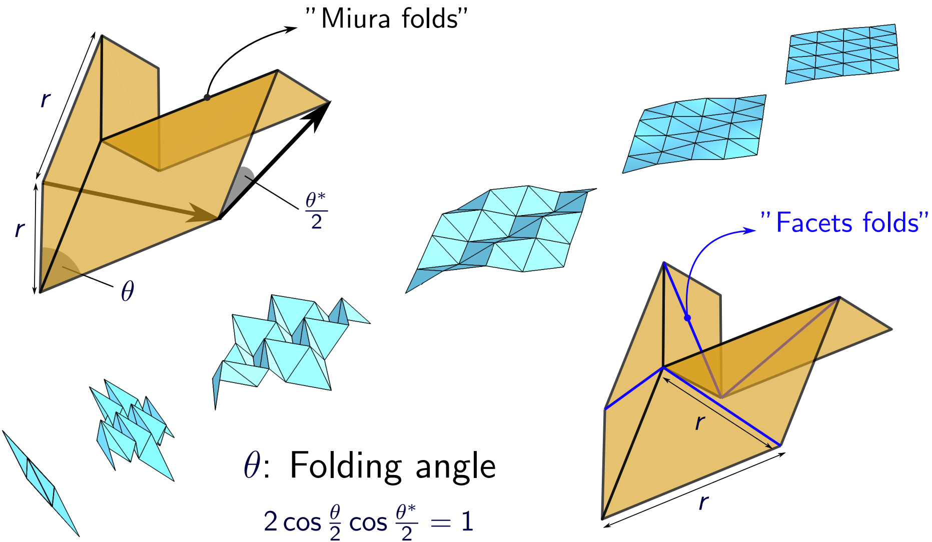


Grey et al. (2018)

$\rightarrow$ Equivalent elastic continuum?

Contents

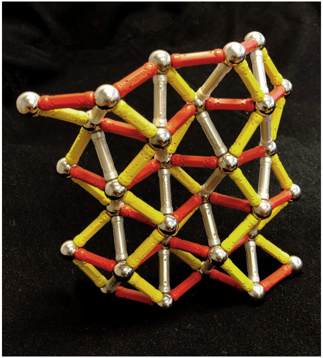
The flexible Miura ori



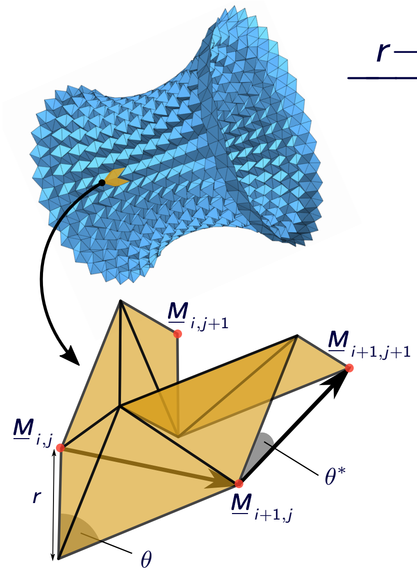
θ: Folding angle

$$2\cos\frac{\theta}{2}\cos\frac{\theta^*}{2} = 1$$

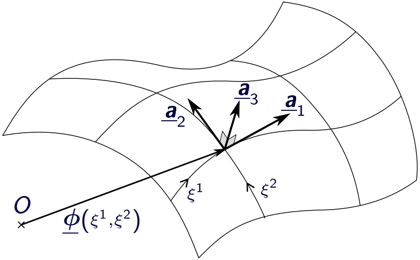
Discrete modeling of the flexible Miura Ori



From discrete to continuous surfaces



$r \to 0$



Smooth $\underline{\phi}\left(\xi^1, \xi^2\right)$?

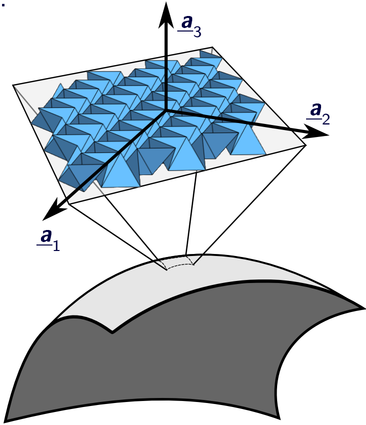
$\Rightarrow$ 1st & 2nd rank finite differences locally independent from $\underline{M}_{i,j}$

$\Rightarrow$ Constrains Metric and Curvature

First rank (metric)

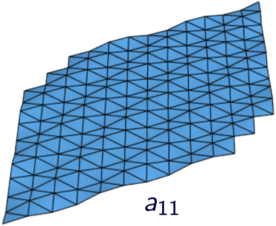
Planar periodic configurations at leading order:

$$\left\{ \begin{array}{l} \underline{\boldsymbol{M}}_{i+1,j} - \underline{\boldsymbol{M}}_{i,j} \\ \underline{\boldsymbol{M}}_{i,j+1} - \underline{\boldsymbol{M}}_{i,j} \end{array} \right. \xrightarrow{r \to 0} \left\{ \begin{array}{l} \partial_1 \underline{\phi} = \underline{\boldsymbol{a}}_1 \\ \partial_2 \underline{\phi} = \underline{\boldsymbol{a}}_2 \end{array} \right.$$



Metric:

$$a_{\alpha\beta}(\theta) = \begin{pmatrix} 4\sin^2 \frac{\theta}{2} & 0 \\ 0 & 4\cos^2 \frac{\theta^*_\theta}{2} \end{pmatrix}$$



Second rank (curvature and...)

Quadratic perturbations at second order:

$$\left\{\begin{array}{l} \underline{\boldsymbol{M}}_{i+1,j} - 2\underline{\boldsymbol{M}}_{i,j} + \underline{\boldsymbol{M}}_{i-1,j} \\ \underline{\boldsymbol{M}}_{i,j+1} - 2\underline{\boldsymbol{M}}_{i,j} + \underline{\boldsymbol{M}}_{i,j-1} \\ \underline{\boldsymbol{M}}_{i,j+1} - 2\underline{\boldsymbol{M}}_{i,j} + \underline{\boldsymbol{M}}_{i+1,j} \end{array}\right. \xrightarrow{r\to 0} \left\{\begin{array}{l} \partial_{11}\underline{\phi} \\ \partial_{22}\underline{\phi} \\ \partial_{12}\underline{\phi} \end{array}\right., \text{ 9 components: } k_{i\alpha\beta} = \underline{\boldsymbol{a}}^i \cdot \partial_{\alpha\beta}\underline{\phi}$$

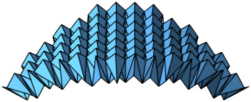
Second rank (curvature and...)

Quadratic perturbations at second order:

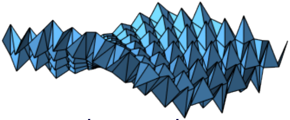
$$\begin{cases} \underline{\boldsymbol{M}}_{i+1,j} - 2\underline{\boldsymbol{M}}_{i,j} + \underline{\boldsymbol{M}}_{i-1,j} \\ \underline{\boldsymbol{M}}_{i,j+1} - 2\underline{\boldsymbol{M}}_{i,j} + \underline{\boldsymbol{M}}_{i,j-1} \\ \underline{\boldsymbol{M}}_{i,j+1} - 2\underline{\boldsymbol{M}}_{i,j} + \underline{\boldsymbol{M}}_{i+1,j} \end{cases} \xrightarrow{r\to 0} \begin{cases} \partial_{11}\underline{\phi} \\ \partial_{22}\underline{\phi} \\ \partial_{12}\underline{\phi} \end{cases}$$

, 9 components: $k_{i\alpha\beta} = \underline{\boldsymbol{a}}^i \cdot \partial_{\alpha\beta}\underline{\phi}$

$$b_{\alpha\beta} = \underline{\boldsymbol{a}}^3 \cdot \partial_{\alpha\beta}\underline{\phi}$$



$k_{311} = b_{11}$



$k_{312} = b_{12}$

Curvature:

$$b_{\alpha\beta}(\theta) = \begin{pmatrix} N\cos^2\theta & M \\ M & -N\cos^2\theta^*_\theta \end{pmatrix}$$

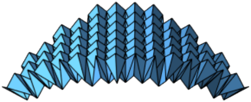
Second rank (curvature and...)

Quadratic perturbations at second order:

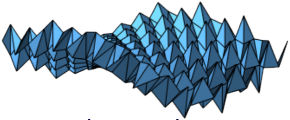
$$\begin{cases} \underline{\boldsymbol{M}}_{i+1,j} - 2\underline{\boldsymbol{M}}_{i,j} + \underline{\boldsymbol{M}}_{i-1,j} \\ \underline{\boldsymbol{M}}_{i,j+1} - 2\underline{\boldsymbol{M}}_{i,j} + \underline{\boldsymbol{M}}_{i,j-1} \\ \underline{\boldsymbol{M}}_{i,j+1} - 2\underline{\boldsymbol{M}}_{i,j} + \underline{\boldsymbol{M}}_{i+1,j} \end{cases} \xrightarrow{r \to 0} \begin{cases} \partial_{11}\underline{\phi} \\ \partial_{22}\underline{\phi} \\ \partial_{12}\underline{\phi} \end{cases}$$

, 9 components: $k_{i\alpha\beta} = \underline{\boldsymbol{a}}^i \cdot \partial_{\alpha\beta}\underline{\phi}$

$b_{\alpha\beta} = \underline{\boldsymbol{a}}^3 \cdot \partial_{\alpha\beta}\underline{\phi}$

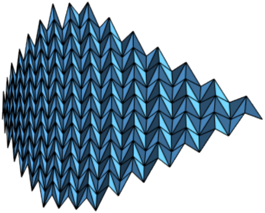


$k_{311} = b_{11}$

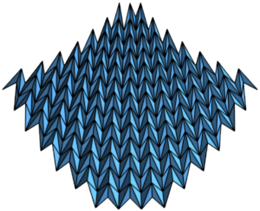


$k_{312} = b_{12}$

$\Gamma^{\gamma}_{\alpha\beta} = \underline{\boldsymbol{a}}^{\gamma} \cdot \partial_{\alpha\beta}\underline{\phi}$



$k_{111} = \Gamma^1_{11}$



$k_{112} = \Gamma^1_{12}$

The continuous fitting problem for the Miura ori

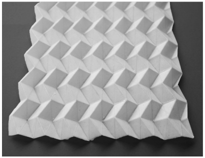
Find $\underline{\phi}$ and $\{\theta(\xi^\alpha), M(\xi^\alpha), N(\xi^\alpha)\}$ such that:

$$a_{\alpha\beta}(\theta) = \begin{pmatrix} 4\sin^2\theta & 0 \\ 0 & 4\cos^2\theta^*_\theta \end{pmatrix} \quad \text{and} \quad b_{\alpha\beta}(\theta) = \begin{pmatrix} N\cos^2\theta & M \\ M & -N\cos^2\theta^*_\theta \end{pmatrix}$$

Only if $\{\theta, M, N\}$ comply with Gauss-Codazzi-Mainardi equations!

Miura Ori:

$$\Rightarrow \quad \frac{\partial_{11}\underline{\phi}}{\cos^2\theta} + \frac{\partial_{22}\underline{\phi}}{\cos^2\theta^*_\theta} = \underline{\mathbf{0}}$$



The continuous fitting problem for the Miura ori

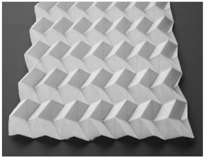
Find $\underline{\phi}$ and $\{\theta(\xi^\alpha), M(\xi^\alpha), N(\xi^\alpha)\}$ such that:

$$a_{\alpha\beta}(\theta) = \begin{pmatrix} 4\sin^2\theta & 0 \\ 0 & 4\cos^2\theta^*_\theta \end{pmatrix} \quad \text{and} \quad b_{\alpha\beta}(\theta) = \begin{pmatrix} N\cos^2\theta & M \\ M & -N\cos^2\theta^*_\theta \end{pmatrix}$$

Only if $\{\theta, M, N\}$ comply with Gauss-Codazzi-Mainardi equations!

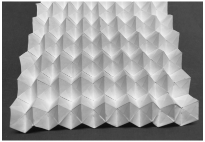
Miura Ori:

$$\Rightarrow \quad \frac{\partial_{11}\underline{\phi}}{\cos^2\theta} + \frac{\partial_{22}\underline{\phi}}{\cos^2\theta^*_\theta} = \underline{\mathbf{0}}$$



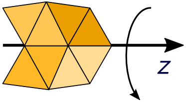
Eggbox Pattern:

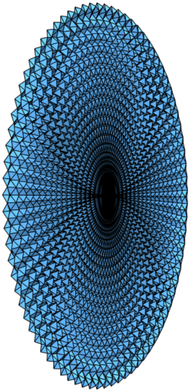
$$\Rightarrow \quad \frac{\partial_{11}\underline{\phi}}{\cos^2\theta} - \frac{\partial_{22}\underline{\phi}}{\cos^2\theta^*_\theta} = \underline{\mathbf{0}}$$



Some closed form illustrations of Miura surfaces

Axisymmetric I

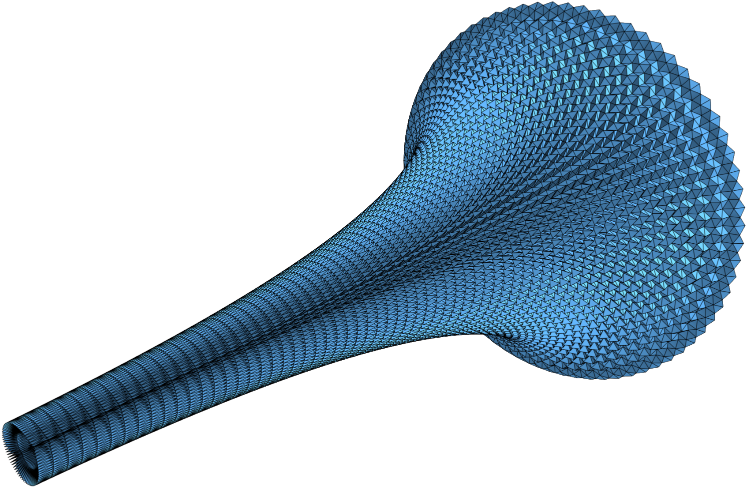




Const. neg. Gauss curvature

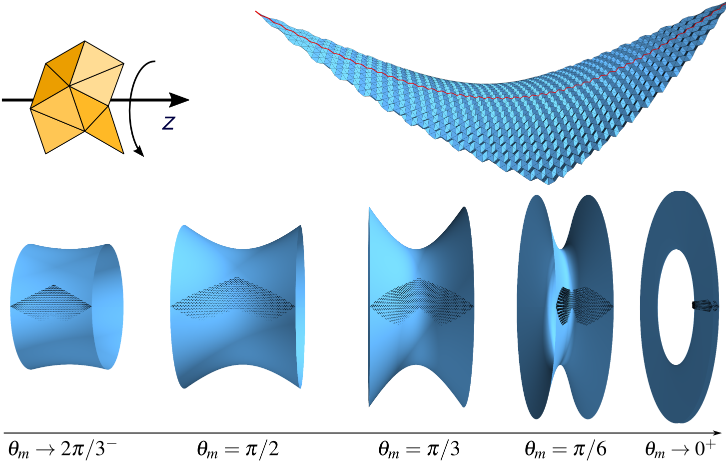
Some closed form illustrations of Miura surfaces

The pseudo sphere



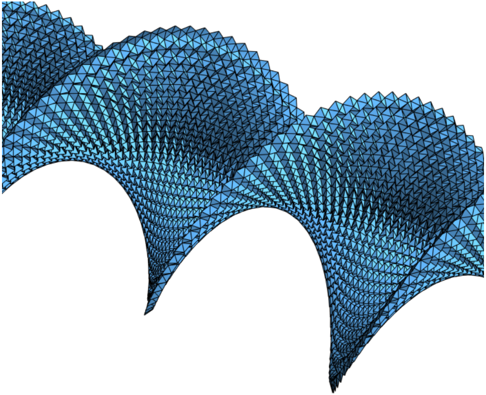
Some closed form illustrations of Miura surfaces

Miura Hyperboloid



Some closed form illustrations of Miura surfaces

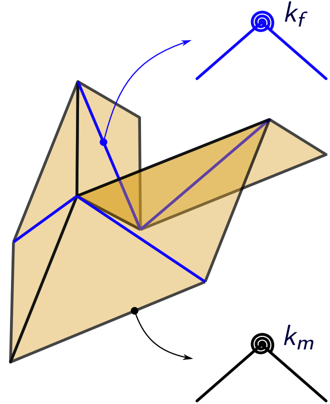
Ruled surfaces



Helicoid

Contents

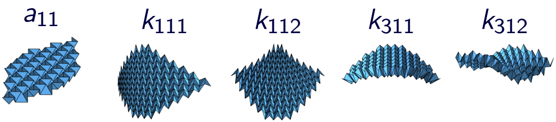
From discrete to continuous elasticity



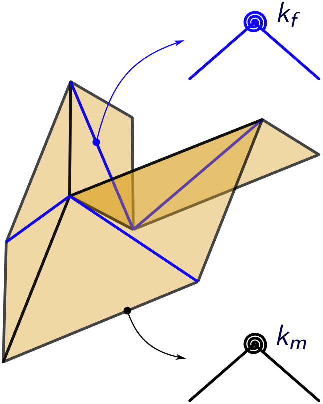
Strain energy density:

$$W^{\text{continuous}} = \frac{1}{r^2} \sum_i \frac{1}{2} k_i (\theta_i - \theta_i^0)^2$$

θ_i are functions of:

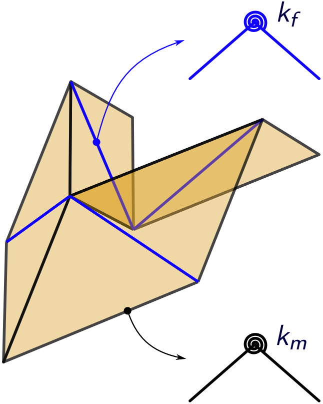


From discrete to continuous elasticity



At leading order when $r \to 0$:

From discrete to continuous elasticity



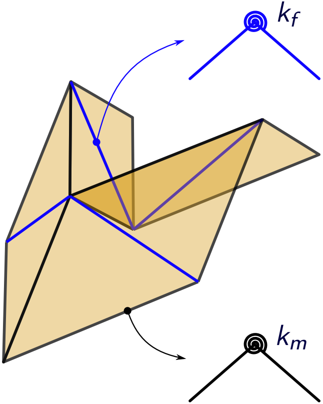
At leading order when $r \to 0$:

- if $k_m > 0$ and $k_f \sim k_m$:

$$W^{\text{continuous}}(a_{11})$$

$\to$ Membrane continuum

From discrete to continuous elasticity



At leading order when $r \to 0$:

- if $k_m > 0$ and $k_f \sim k_m$:

  $$W^{\text{continuous}}(a_{11})$$

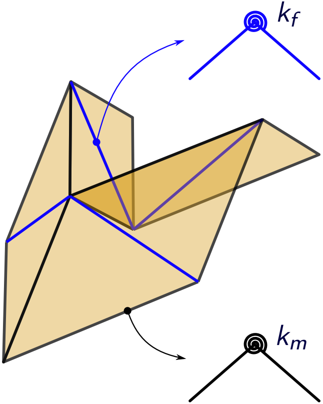
  $\to$ Membrane continuum

- if $k_m = 0$ and $k_f > 0$:

  $$W^{\text{continuous}}(k_{111}, k_{112}, k_{311}, k_{312}; a_{11})$$

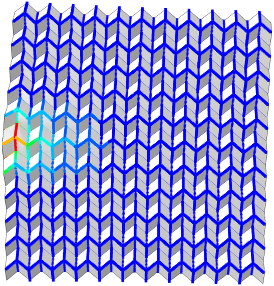
  $\to$ Shell (out-of-plane) and Strain-gradient (in-plane) continuum

From discrete to continuous elasticity



When $k_m > 0$ and $k_m \ll k_f$:

charac. length: $l \sim r\sqrt{\frac{k_f}{k_m}}$

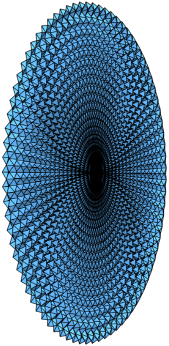


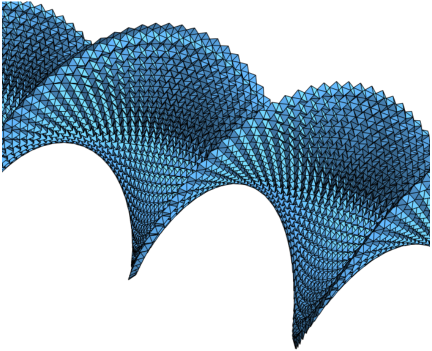
Grey et al. (2018)

Conclusion

- Miura Ori surfaces
- 2 finite deformation continua:
  - Membrane
  - Shell + Strain-gradient

- FEM implementation?
- Experiments?





References

Dudte, L. H., Vouga, E., Tachi, T., Mahadevan, L., 2016. Programming curvature using origami tessellations. Nature Materials (January), 1–24.

Filipov, E. T., Tachi, T., Paulino, G. H., 2015. Origami tubes assembled into stiff, yet reconfigurable structures and metamaterials. Proceedings of the National Academy of Sciences of the United States of America 112 (40), 12321–12326.

Grey, S. W., Schenk, M., Scarpa, F. L., 2018. Local Actuation of Tubular Origami. The proceedings from the seventh meeting of Origami, Science, Mathematics and Education (October).

Lebée, A., 2010. Thick periodic plates homogenization, application to sandwich panels including chevron folded core. Ph.D. thesis, Université Paris-Est.

Lebée, A., jun 2015. From Folds to Structures, a Review. International Journal of Space Structures 30 (2), 55–74.

Miura, K., apr 1993. Concepts of Deployable Space Structures. International Journal of Space Structures 8 (1-2), 3–16.

Resch, R. D., Christiansen, H., 1970. The design and analysis of kinematic folded-plate systems. In: Proceedings of the Symposium for folded plates and prismatic structures, International association for shell structures. Vienna, pp. 1–36.